Agnostic ringdown tests of general relativity

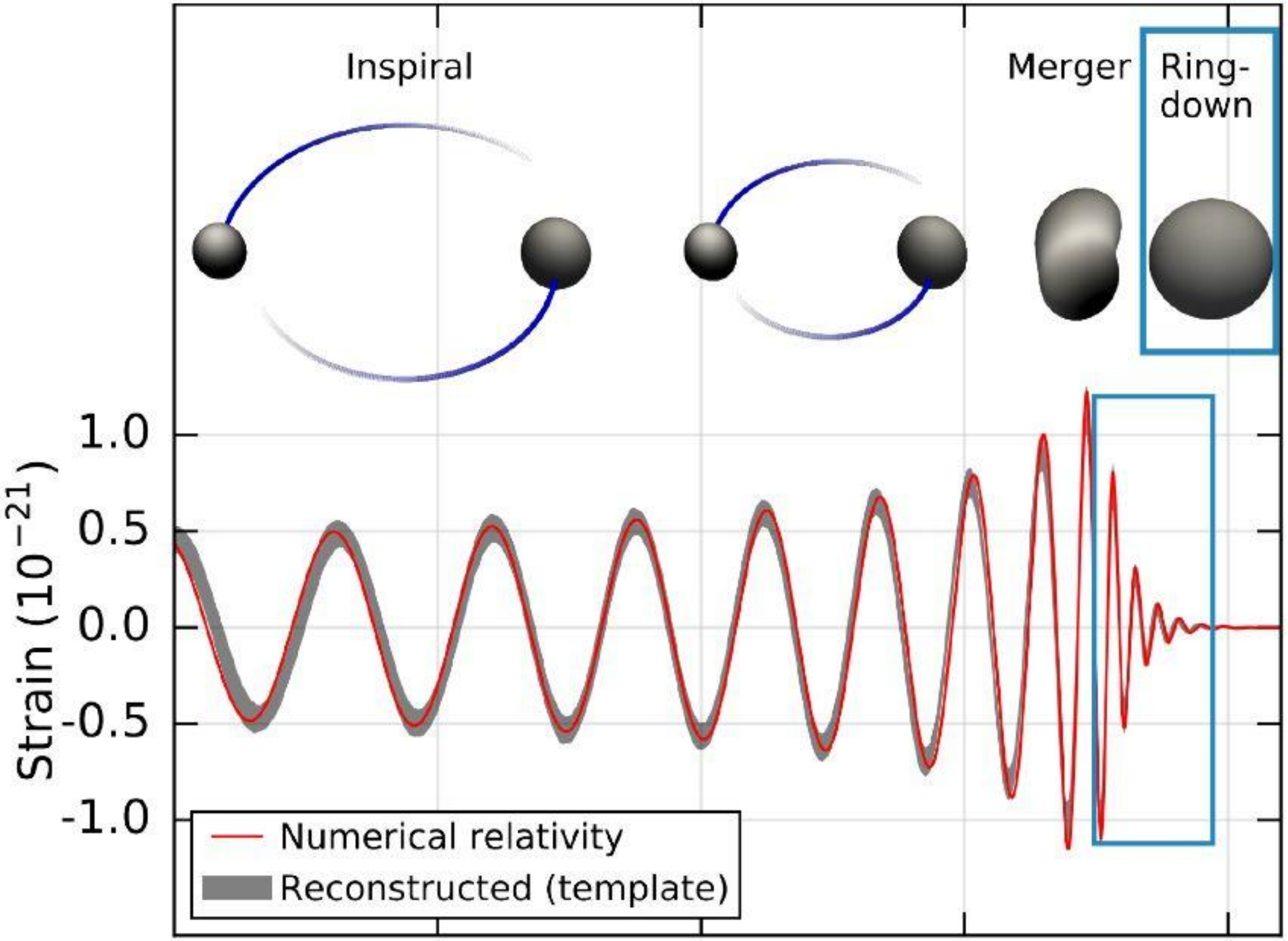
Nicola Franchini

CENTRA, Instituto Superior Tecnico, Lisbon

Gravity group meeting - SISSA

29 Jan 2025

Confrontation between theory and observations

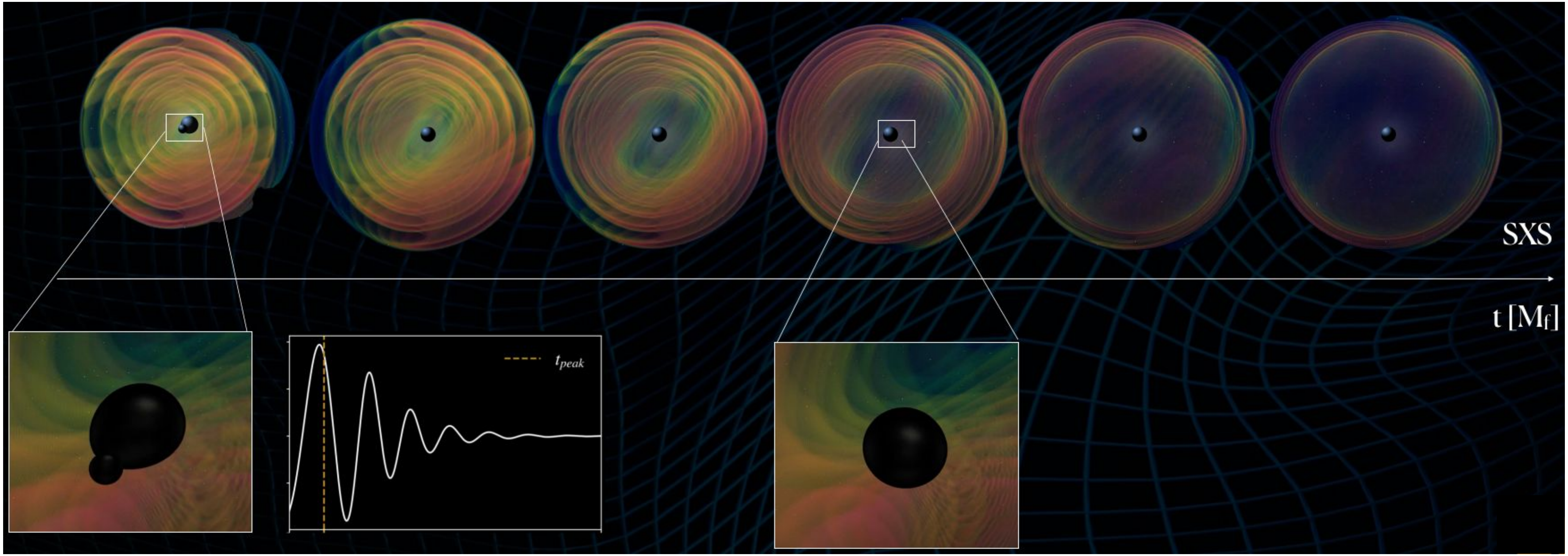


Credits: LIGO-Virgo collaboration 2016

Confrontation between theory and observations

From numerical relativity simulation:

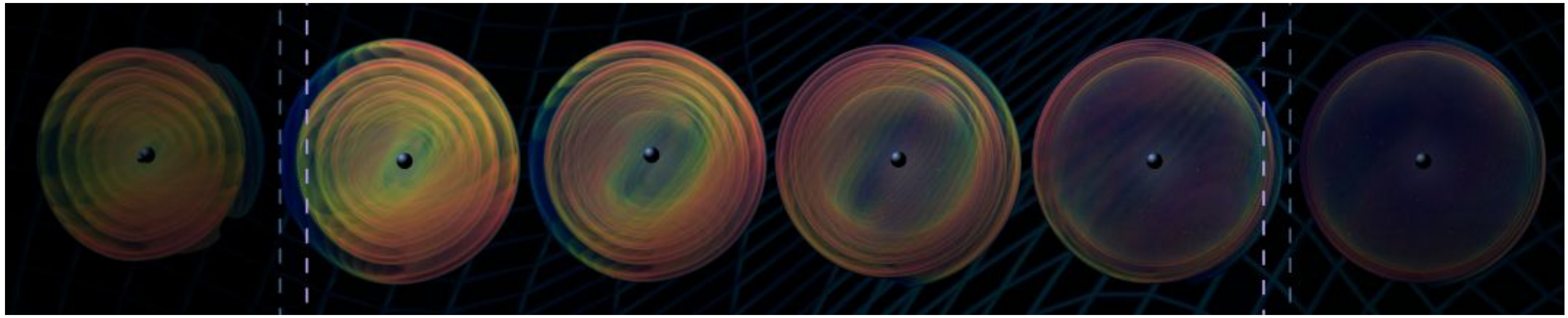
Dynamical horizon formation... ...followed by quasi-normal mode ringing



Courtesy of Giada Caneva Santoro

Confrontation between theory and observations

Perturbative scenario



$$h_+ - \mathrm{i}h_\times = \sum_j A_j \mathrm{e}^{\mathrm{i}\omega_j t + \phi_j} \mathrm{e}^{-(t-t_0)/\tau_j}$$

Phase

Starting time

Amplitude

Frequency

Damping time

Confrontation between theory and observations

Overview of LIGO-Virgo-KAGRA (LVK) ringdown analysis:

	Inspiral-Merger-Ringdown	Ringdown only
Pros	• Longer signal • Higher SNR	• Best test of *no-hair theorems* • Easy modeling (sum of damped sinusoids) • Information on progenitors (from amplitude and phases)
Cons	• Lot of parameters • Hard to disentangle RD-only features • Time-consuming analysis	• Lower SNR • Hard to define time of start • Information on progenitors (need of fitting NR waveforms)

Confrontation between theory and observations

Overview of LIGO-Virgo-KAGRA (LVK) ringdown analysis:

	Inspiral-Merger-Ringdown	Ringdown only
Pros	• Longer signal • Higher SNR	• Best test of *no-hair theorems* • Easy modeling (sum of damped sinusoids) • Information on progenitors (from amplitude and phases)
Cons	• Lot of parameters • Hard to disentangle RD-only features • Time-consuming analysis	• Lower SNR • Hard to define time of start • Information on progenitors (need of fitting NR waveforms)

Confrontation between theory and observations

Overview of LVK ringdown analysis:

- Time-domain analysis — *pyRing*: Carullo, del Pozzo, Veitch (2019); *ringdown*: Isi, Farr (2021)
- Frequency domain analysis — *pSEOB*: Brito, Buonanno, Raymond (2018); Gosh, Brito, Buonanno (2021); Maggio, *et al* (2022)
- Hybrid mode cleaning — *Rational filters*: Ma, Sun, Chen (2023)

Confrontation between theory and observations

Overview of LVK ringdown analysis:

- Time-domain analysis — *pyRing*: Carullo, del Pozzo, Veitch (2019); *ringdown*: Isi, Farr (2021)
- Frequency domain analysis — *pSEOB*: Brito, Buonanno, Raymond (2018); Gosh, Brito, Buonanno (2021); Maggio, *et al* (2022)
- Hybrid mode cleaning — *Rational filters*: Ma, Sun, Chen (2023)

Bayesian inference

Bayes' theorem

Probability distribution of B given A is true: likelihood

Probability distribution of A: prior distribution

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Probability distribution of A given B is true: posterior distribution

Probability distribution of B: marginal distribution

Time domain analysis

Type of analysis:

- Damped sinusoids

$$h_+ - \mathrm{i}h_\times = \sum_j A_j \mathrm{e}^{\mathrm{i}\omega_j t + \phi_j} \mathrm{e}^{-(t-t_0)/\tau_j}$$

Sampling over: frequencies, damping times, amplitudes and phases

Time domain analysis

Type of analysis:

- Kerr QNMs

$$\omega(M, a)$$

$$h_+ - \mathrm{i}h_\times = \sum_j A_j \mathrm{e}^{\mathrm{i}\omega_j t + \phi_j} \mathrm{e}^{-(t-t_0)/\tau_j}$$

$$\tau(M, a)$$

Sampling over: mass, spin, amplitudes and phases

Time domain analysis

Type of analysis:

- Kerr QNMs + NR fits

$$h_+ - \mathrm{i}h_\times = \sum_j A_j \mathrm{e}^{\mathrm{i}\omega_j t + \phi_j} \mathrm{e}^{-(t-t_0)/\tau_j}$$

$$\omega(M, a)$$

$$\phi(\eta, \chi_s, \chi_p)$$

$$A(\eta, \chi_s, \chi_p)$$

$$\tau(M, a)$$

Sampling over: mass, spin, progenitor masses and spins

Tests of GR with time domain analysis

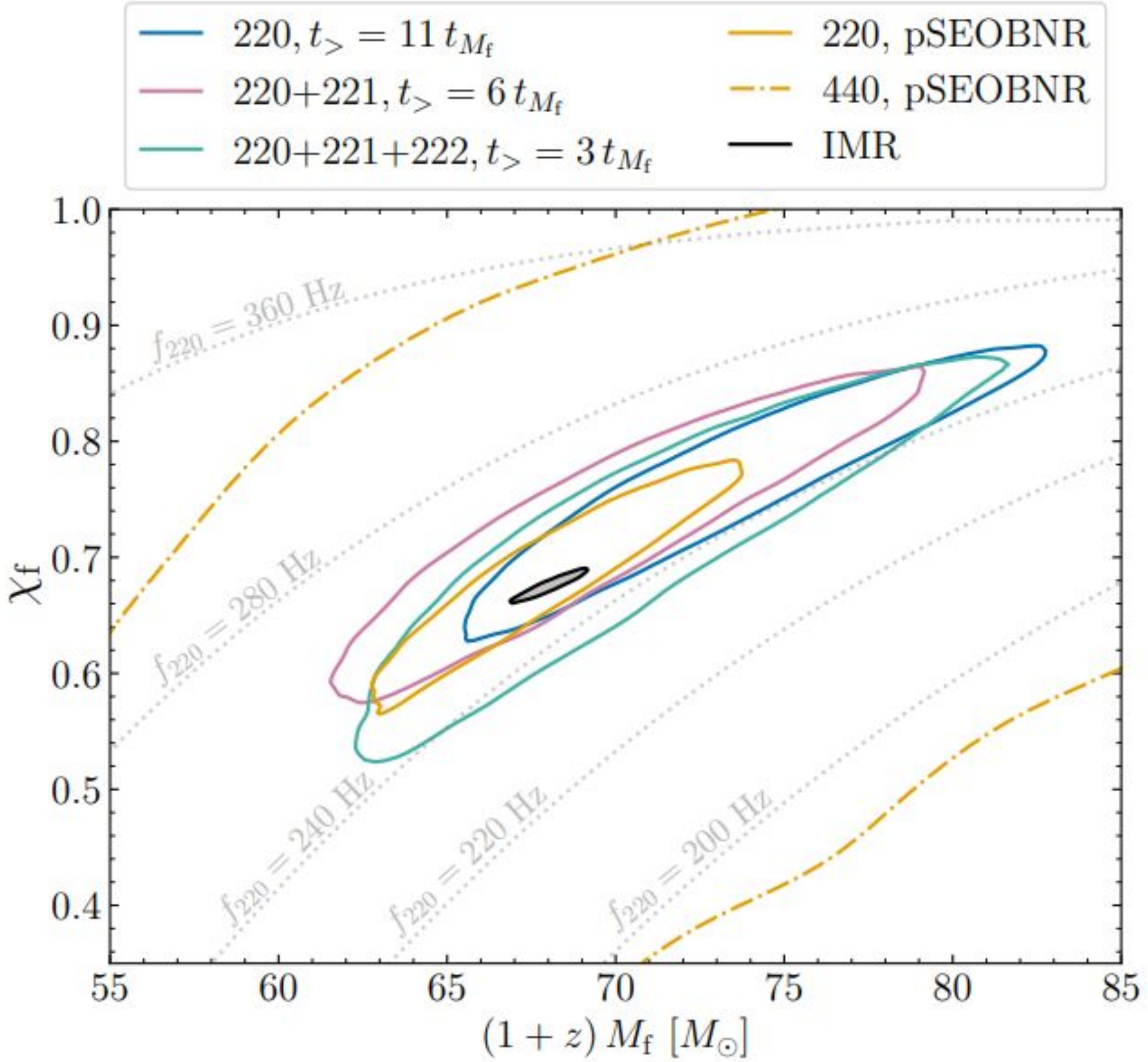
The standard LVK routine

Consistency test

Constrain mass and spin

Compare it against inspiral-merger-ringdown (IMR) predictions

Best constraints for GW250114



LIGO - Virgo - Kagra collaboration 2025

Tests of GR with time domain analysis

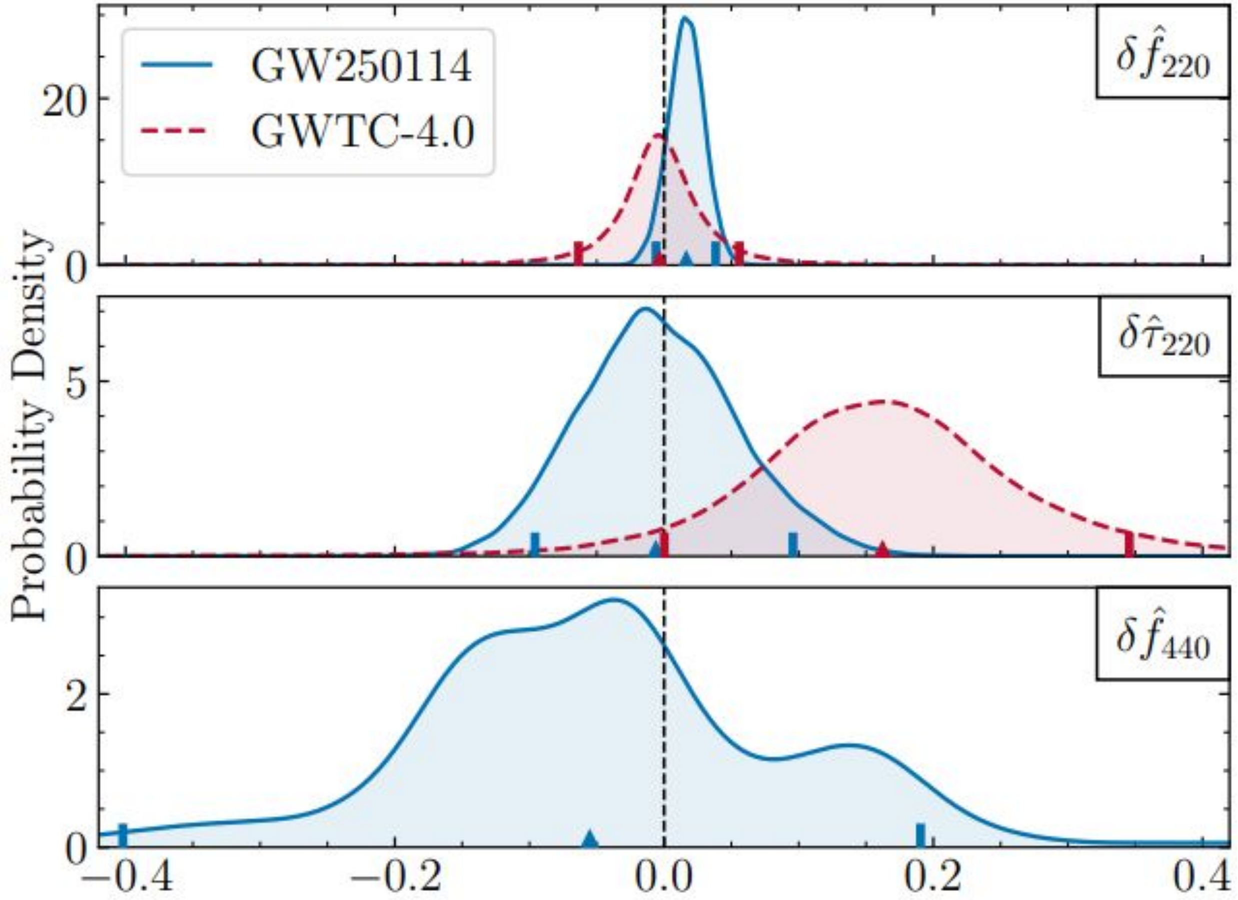
The standard LVK routine

Unmodeled deviations

Constrain deviations of frequencies and damping times

$$\delta \hat{f}_{\ell mn} = \log(f_{\ell mn}/f_{\ell mn,\mathrm{Kerr}})$$
$$\delta \hat{\tau}_{\ell mn} = \log(\tau_{\ell mn}/\tau_{\ell mn,\mathrm{Kerr}})$$

Best constraints for GW250114



LIGO - Virgo - Kagra collaboration 2025

Can we improve ringdown tests of GR?

Current issues:

- Mode detection
- Tests rely on IMR
- Parametrization too general

Can we improve ringdown tests of GR?

Current issues:

- Mode detection
- Tests rely on IMR
- Parametrization too general

Ideally we need:

- Use of all ringdown quantities
- Tests independent from IMR analysis
- Parametrizations that depend on a few parameters
- Connect parameters to dynamics and metric of deviations
- Clear mapping to known theories

Can we improve ringdown tests of GR?

Current issues:

- Mode detection
- Tests rely on IMR
- Parametrization too general

Ideally we need:

- Use of all ringdown quantities
- Tests independent from IMR analysis
- Parametrizations that depend on a few parameters
- Connect parameters to dynamics and metric of deviations
- Clear mapping to known theories

QNM ratios

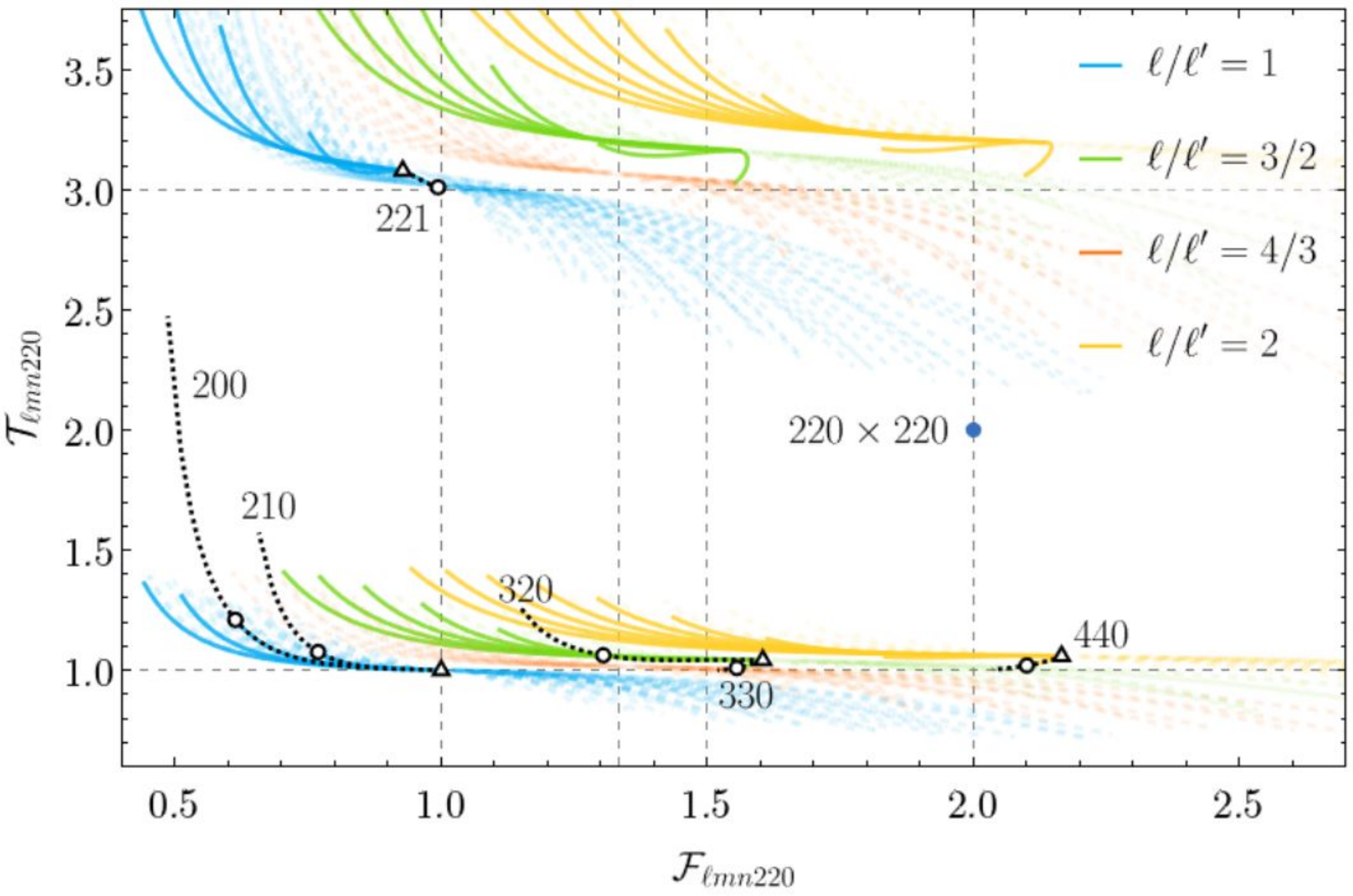
From a DS analysis

Take ratios of freqs and damping times

$$\mathcal{F}_{jj'} = \frac{f_j}{f_{j'}} \qquad \mathcal{T}_{jj'} = \frac{\tau_{j'}}{\tau_j}$$

GR predictions narrow:

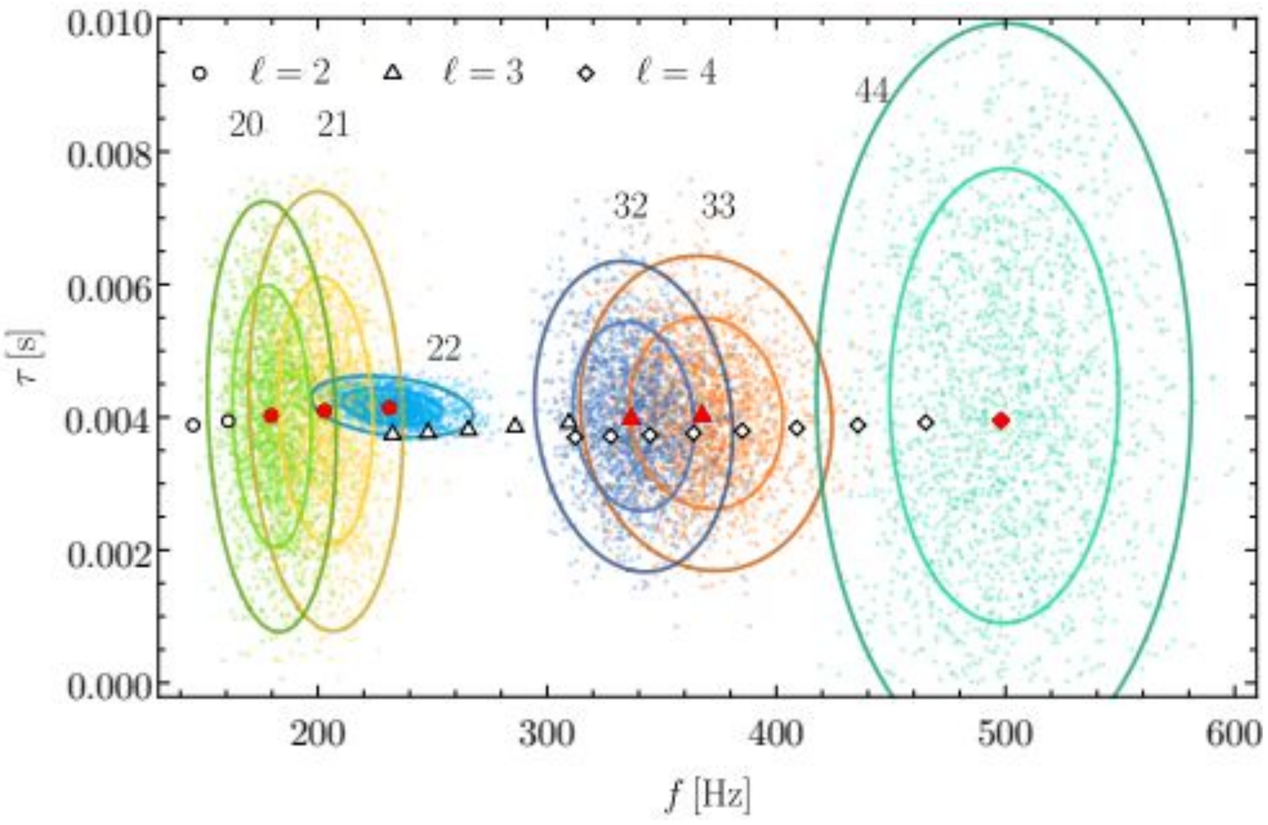
- No mass dependency
- Little spin dependency

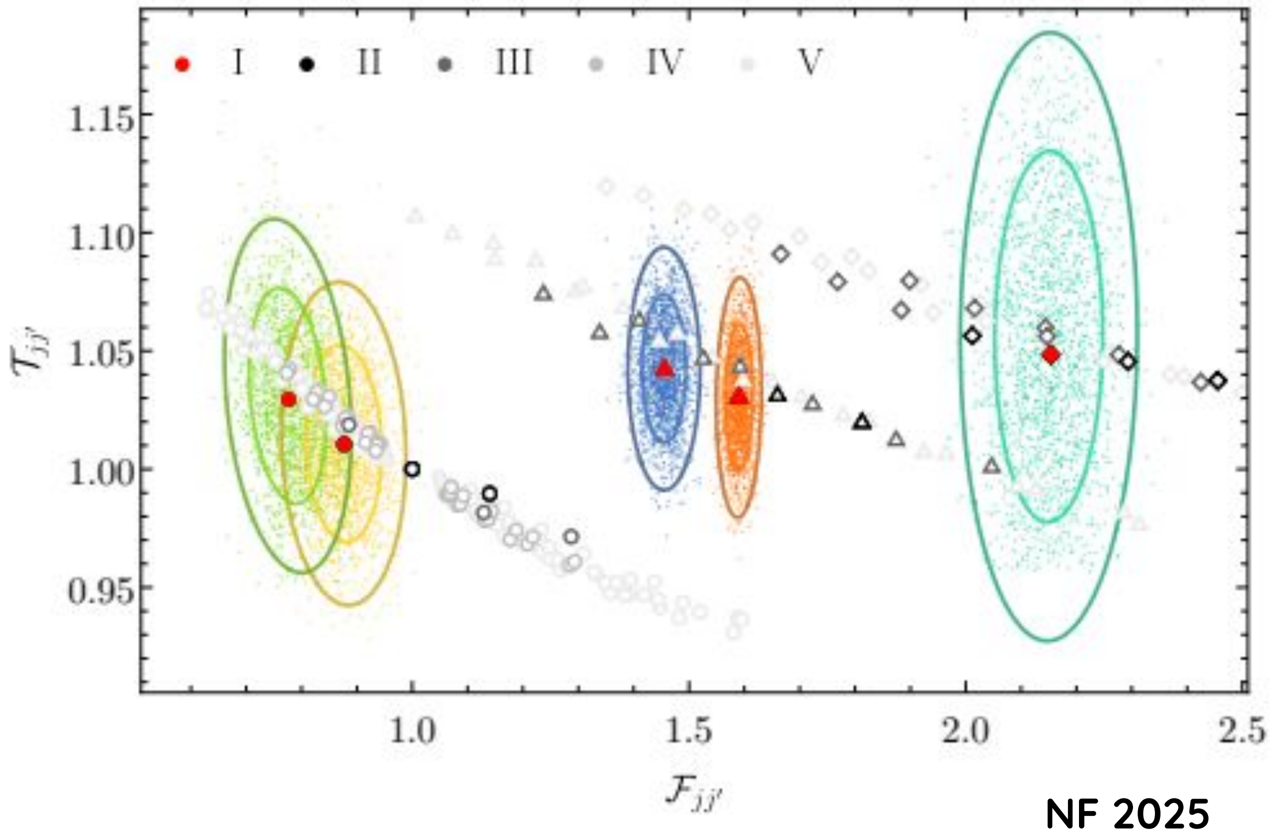


QNM ratios

Improvements against standard DS analysis

For a (2,2)+(l,m) injection in ET [Bhagwat, Pacilio, Pani, Mapelli 2023]





NF 2025

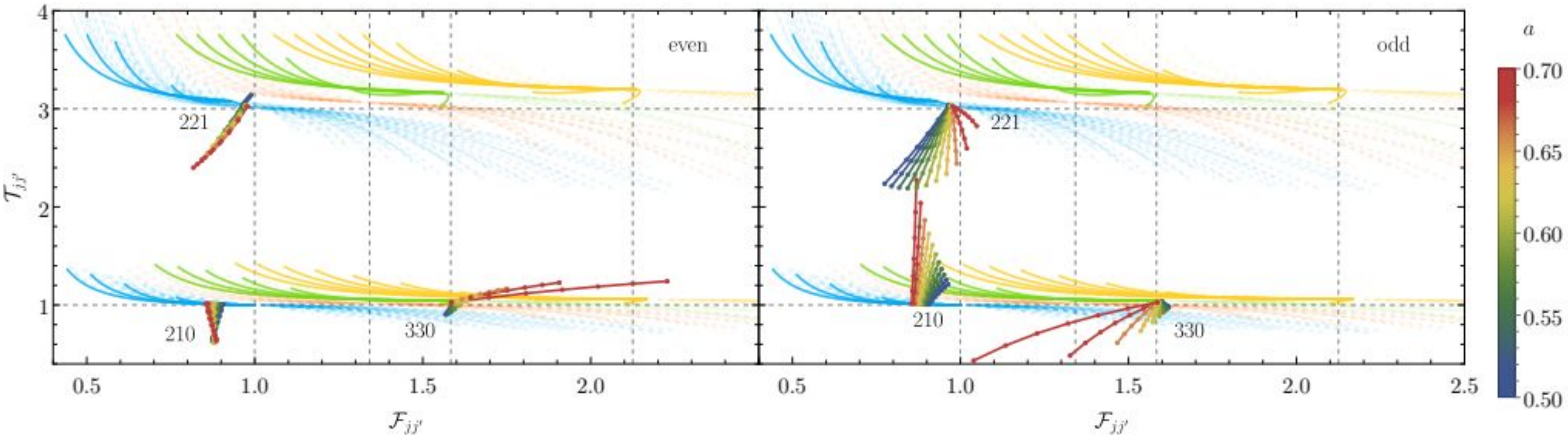
QNM ratios as tests of GR

For Higher-Derivative Gravity (later more details on the computation)

$$S_{\text{EFT}} = \frac{1}{16\pi}\int d^4x\sqrt{|g|}\left[R + \ell^4\left(\lambda_{\text{ev}}\mathcal{R}^3 + \lambda_{\text{odd}}\tilde{\mathcal{R}}^3\right)\right]$$

$$\mathcal{R}^3 = R_{\mu\nu}{}^{\rho\sigma}R_{\rho\sigma}{}^{\delta\gamma}R_{\delta\gamma}{}^{\mu\nu}$$

$$\tilde{\mathcal{R}}^3 = R_{\mu\nu}{}^{\rho\sigma}R_{\rho\sigma}{}^{\delta\gamma}\tilde{R}_{\delta\gamma}{}^{\mu\nu}$$



NF 2025

Amplitude-phase consistency

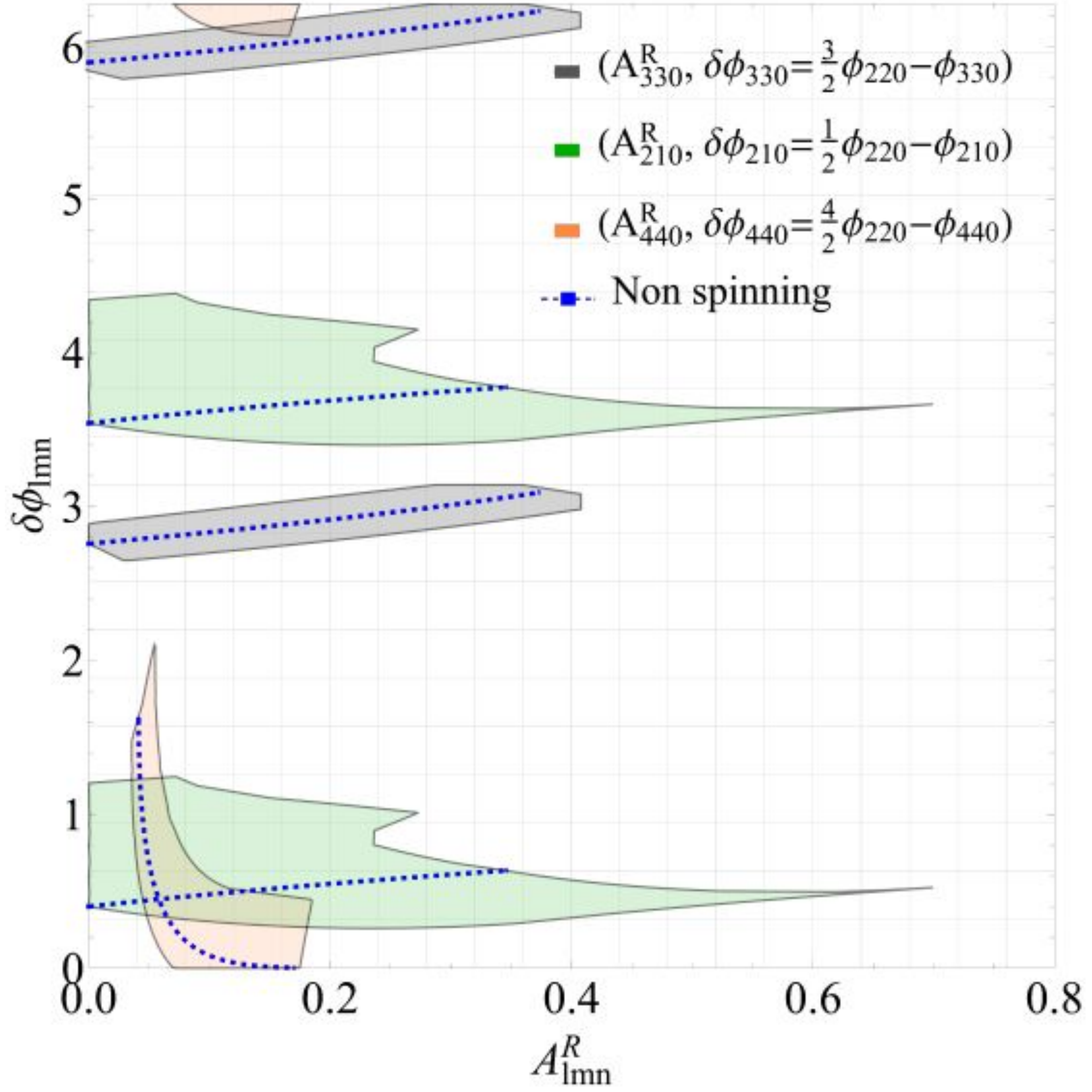
Use info from amp and phase

Method developed in [Jimenez-forteza, Bhagwat, Kumar, Pani, 2022]

Constrain

$$\overline{A}_{\ell mn} = \frac{A_{\ell mn}}{A_{220}}$$

$$\delta\phi_{\ell mn} = \frac{m}{2}\phi_{220} - \phi_{\ell mn}$$



Amplitude-phase consistency

Use info from amp and phase

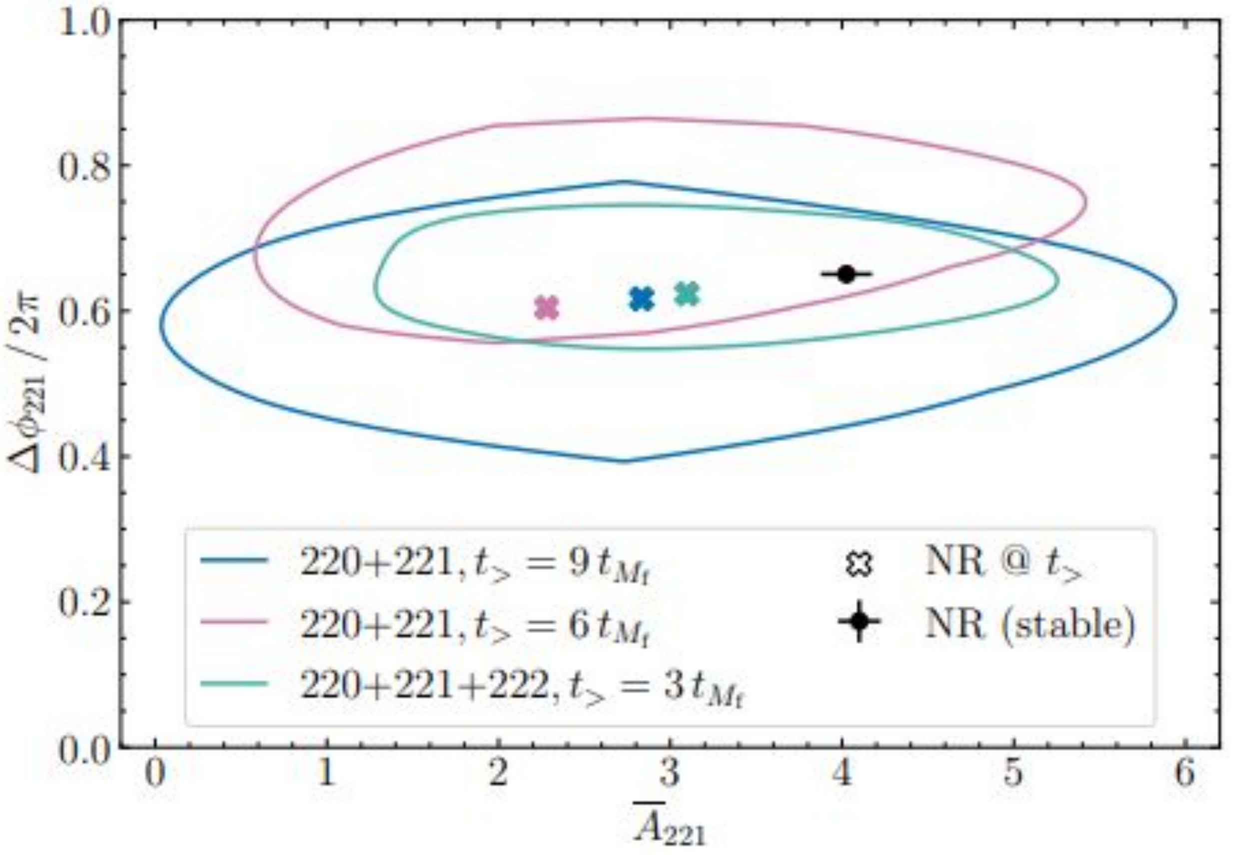
Method developed in [Jimenez-forteza, Bhagwat, Kumar, Pani, 2022]

Constrain

$$\overline{A}_{\ell mn} = \frac{A_{\ell mn}}{A_{220}}$$

$$\delta\phi_{\ell mn} = \frac{m}{2}\phi_{220} - \phi_{\ell mn}$$

Best constraints for GW250114



LIGO - Virgo - Kagra collaboration 2025

QNM ratios & amp-phase consistency

Advantages:

- Makes use of all ringdown quantities
- (almost) independent from IMR analysis
- Posteriors don't depend on mass of the remnant, mildly on the spin
- Posteriors depend on progenitors (inspiral-only consistency check is possible)
- Has a clear behaviour for GR deviations
- Hierarchical analysis with multiple events is natural

Can we improve ringdown tests of GR?

Current issues:

- Mode detection
- Tests rely on IMR
- Parametrization too general

Ideally we need:

- Use of all ringdown quantities
- Tests independent from IMR analysis
- Parametrizations that depend on a few parameters
- Connect parameters to dynamics and metric of deviations
- Clear mapping to known theories

Maselli, Pani, Gualtieri, Berti 2019;

Mass-spin parametrization: ParSpec

$$f_{\ell mn} = \frac{1}{M} \sum_{k=0}^{K} \chi^k f_{\ell mn}^{(k)} \left(1 + \gamma\, \delta f_{\ell mn}^{(k)}\right) \qquad \tau_{\ell mn} = M \sum_{k=0}^{K} \chi^k \tau_{\ell mn}^{(k)} \left(1 + \gamma\, \delta \tau_{\ell mn}^{(k)}\right)$$

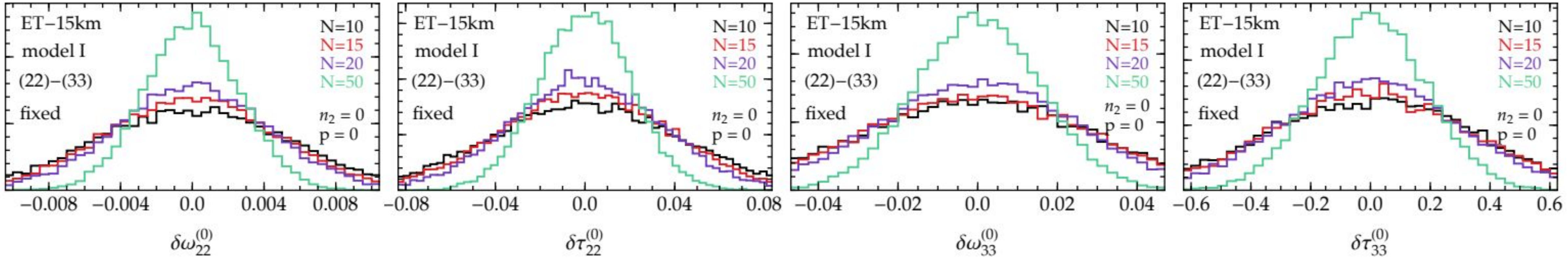
- Double expansion: spin and coupling constant
- Easy mapping to known theories
- Captures physical behaviour of deviations
- Introduces a large number of parameters: prone to degeneracies

Maselli, Pani, Gualtieri, Berti 2019;
Maselli, Yi, et al 2024

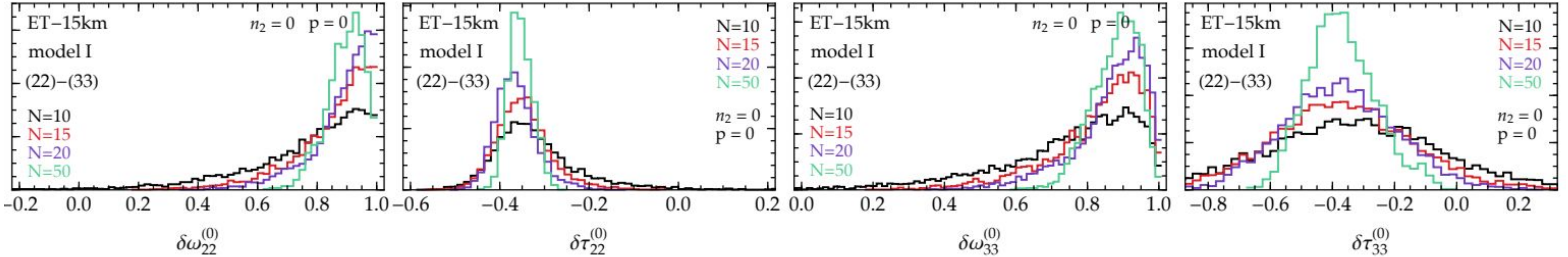
Mass-spin parametrization: ParSpec

$$f_{\ell mn} = \frac{1}{M}\sum_{k=0}^{K} \chi^k f^{(k)}_{\ell mn}\left(1+\gamma\,\delta f^{(k)}_{\ell mn}\right) \qquad \tau_{\ell mn} = M\sum_{k=0}^{K} \chi^k \tau^{(k)}_{\ell mn}\left(1+\gamma\,\delta \tau^{(k)}_{\ell mn}\right)$$

ET prospects, two modes, one dimensionless deviation



Adding mass and spin shifts



Agnostic effective potential

Parametrized ringdown formalism - nonrotating case

Cardoso et al 2019;
McManus et al 2019;
Volkel, NF, Barausse 2022.

A modified master equation...

$$f\frac{\mathrm{d}}{\mathrm{d}r}\left(f\frac{\mathrm{d}\mathbf{\Phi}}{\mathrm{d}r}\right)+\left[\omega^2-f\mathbf{V}\right]\mathbf{\Phi}=0$$

$$f=1-r_+/r$$

$$V_{ij}=V_{ij}^{\mathrm{GR}}+\delta V_{ij},$$

$$\delta V_{ij}=\frac{1}{r_+^2}\sum_{k=0}^{\infty}\alpha_{ij}^{(k)}\left(\frac{r_+}{r}\right)^k$$

Agnostic effective potential

Parametrized ringdown formalism - nonrotating case

Cardoso et al 2019;
McManus et al 2019;
Volkel, NF, Barausse 2022.

A modified master equation...

$$f\frac{\mathrm{d}}{\mathrm{d}r}\left(f\frac{\mathrm{d}\mathbf{\Phi}}{\mathrm{d}r}\right)+\left[\omega^2-f\mathbf{V}\right]\mathbf{\Phi}=0$$

$$f=1-r_+/r$$

$$V_{ij}=V_{ij}^{\mathrm{GR}}+\delta V_{ij},$$

$$\delta V_{ij}=\frac{1}{r_+^2}\sum_{k=0}^{\infty}\alpha_{ij}^{(k)}\left(\frac{r_+}{r}\right)^k$$

...yields to corrected modes

$$\omega\approx\omega^0+\alpha_{ij}^{(k)}d_{(k)}^{ij}$$

$$+\frac{1}{2}\alpha_{ij}^{(k)}\alpha_{pq}^{(s)}e_{(ks)}^{ijpq}$$

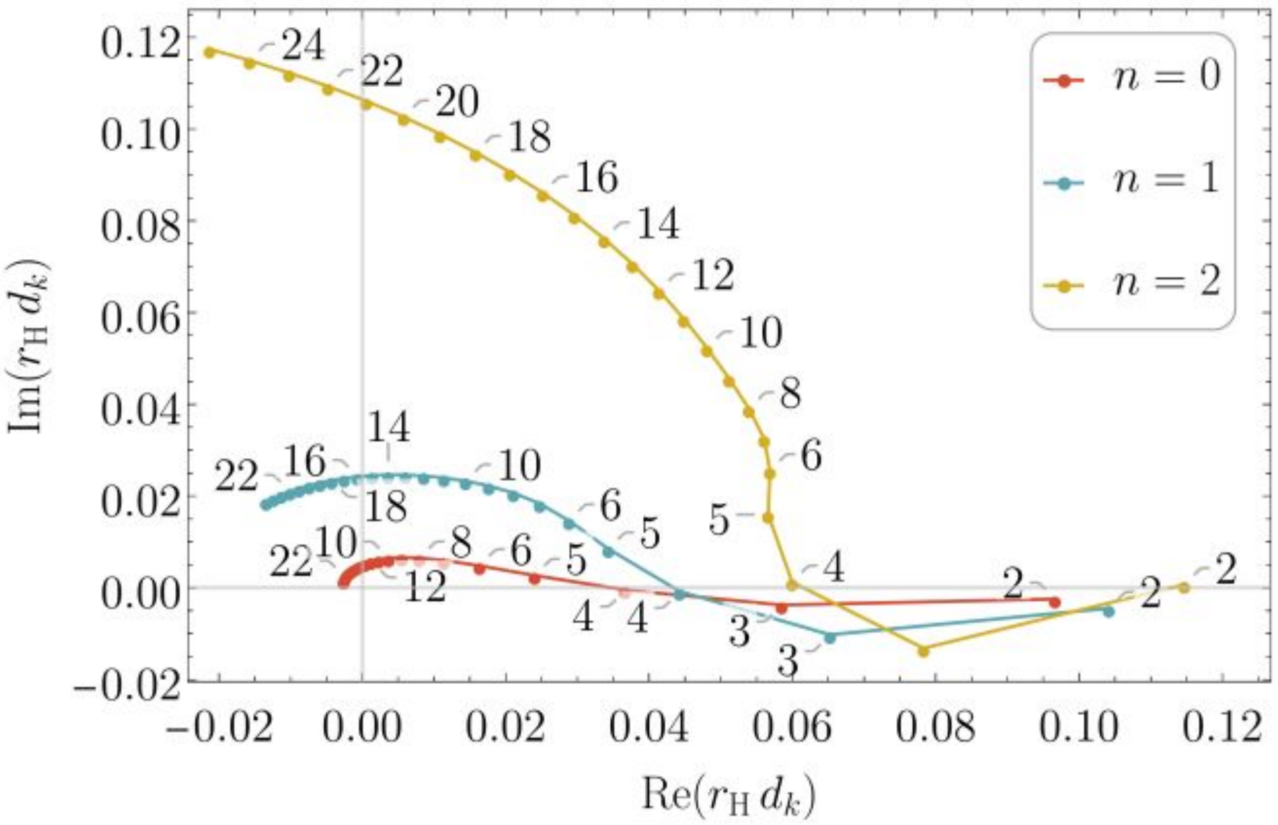
Agnostic effective potential

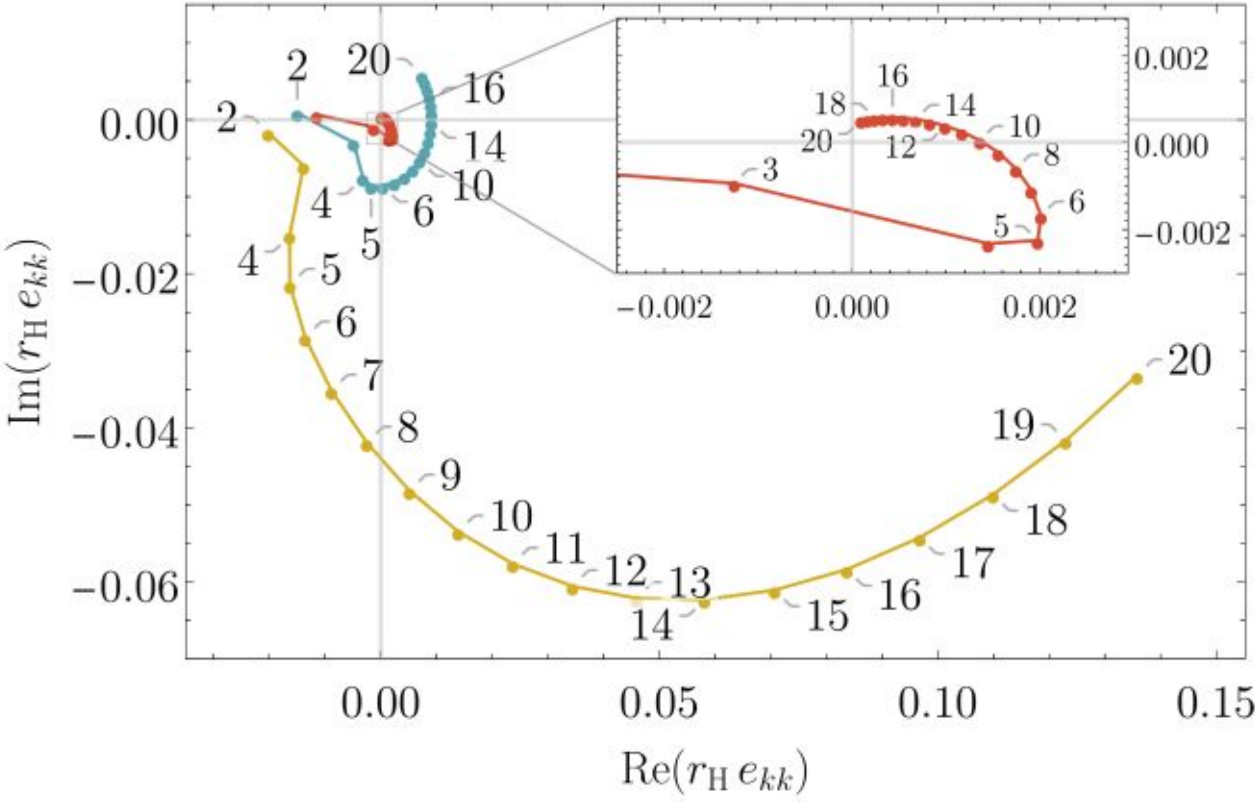
Parametrized ringdown formalism - nonrotating case

Cardoso et al 2019;
McManus et al 2019;
Volkel, NF, Barausse 2022.

$$\omega \approx \omega^0 + \alpha_{ij}^{(k)} d_{(k)}^{ij} + \frac{1}{2}\alpha_{ij}^{(k)}\alpha_{pq}^{(s)} e_{(ks)}^{ijpq}$$

Coefficients available on [github]





Volkel, NF, Barausse 2022
Volkel, NF, Barausse, Berti 2022

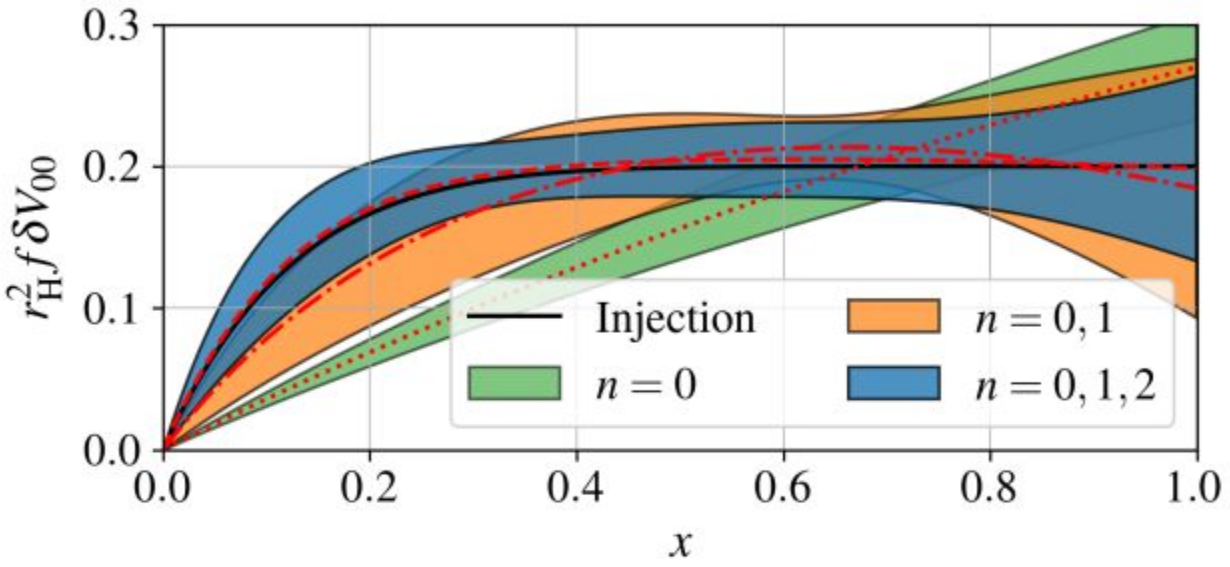
Agnostic effective potential

Parametrized ringdown formalism - nonrotating case

Potential reconstruction with principal component analysis

- 8 deviation parameters injected
- 1 to 3 modes recovered

$$\omega \approx \omega^0 + \alpha_{ij}^{(k)} d_{(k)}^{ij}$$



Volkel, NF, Barausse 2022
Volkel, NF, Barausse, Berti 2022

Agnostic effective potential

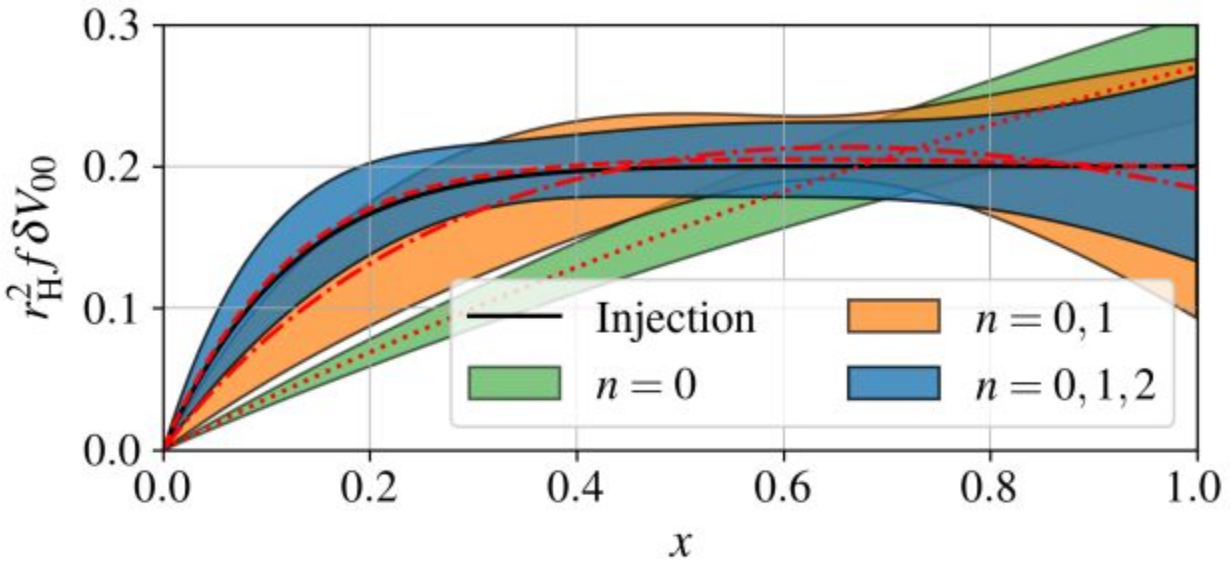
Parametrized ringdown formalism - nonrotating case

Potential reconstruction with principal component analysis

- 8 deviation parameters injected
- 1 to 3 modes recovered

$$\omega \approx \omega^0 + \alpha_{ij}^{(k)} d^{ij}_{(k)}$$

What about the constraints on each deviation parameter?



Volkel, NF, Barausse 2022
Volkel, NF, Barausse, Berti 2022

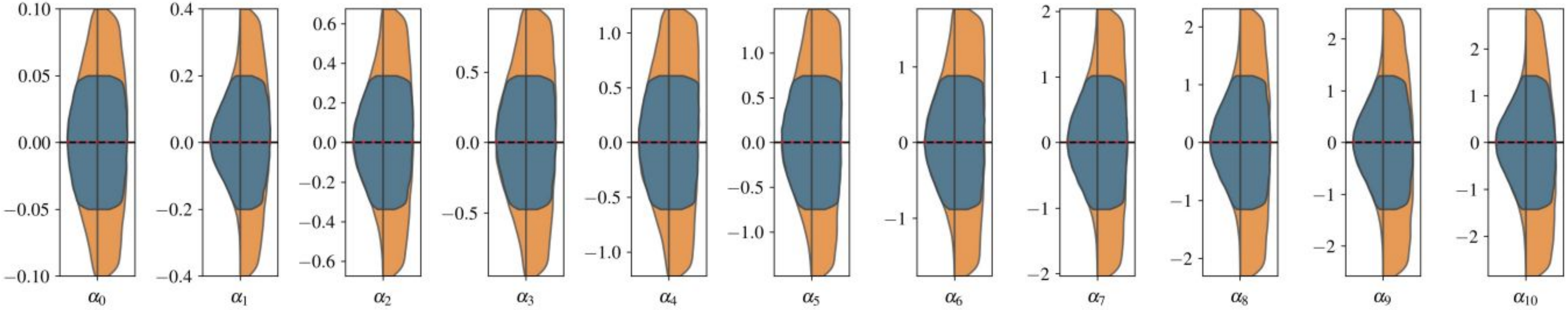
Agnostic effective potential

Parametrized ringdown formalism - nonrotating case

$$\omega \approx \omega^0 + \alpha_{ij}^{(k)} d^{ij}_{(k)}$$

MCMC potential reconstruction

- GR injection (red line)
- Left: one value varied at time; right: all values varied
- 2 modes recovered



Volkel, NF, Barausse 2022
Volkel, NF, Barausse, Berti 2022

Agnostic effective potential

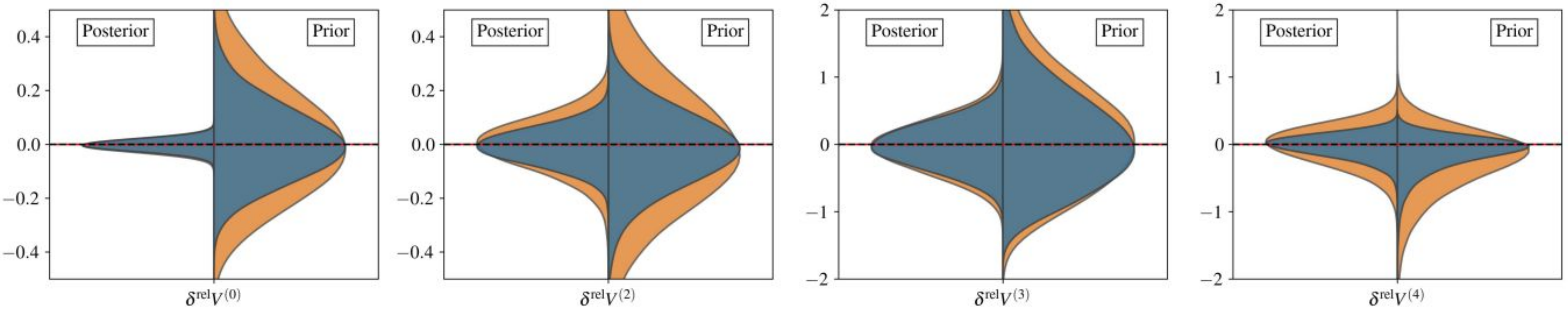
Parametrized ringdown formalism - nonrotating case

$$\omega_n^2 = V^{(0)} - \mathrm{i}\sqrt{-2V^{(2)}}\left(n + \frac{1}{2}\right)$$

Schutz, Will 1985

MCMC potential reconstruction

- GR injection (red line)
- WKB-inspired: constrain potential at the peak
- 2 modes recovered

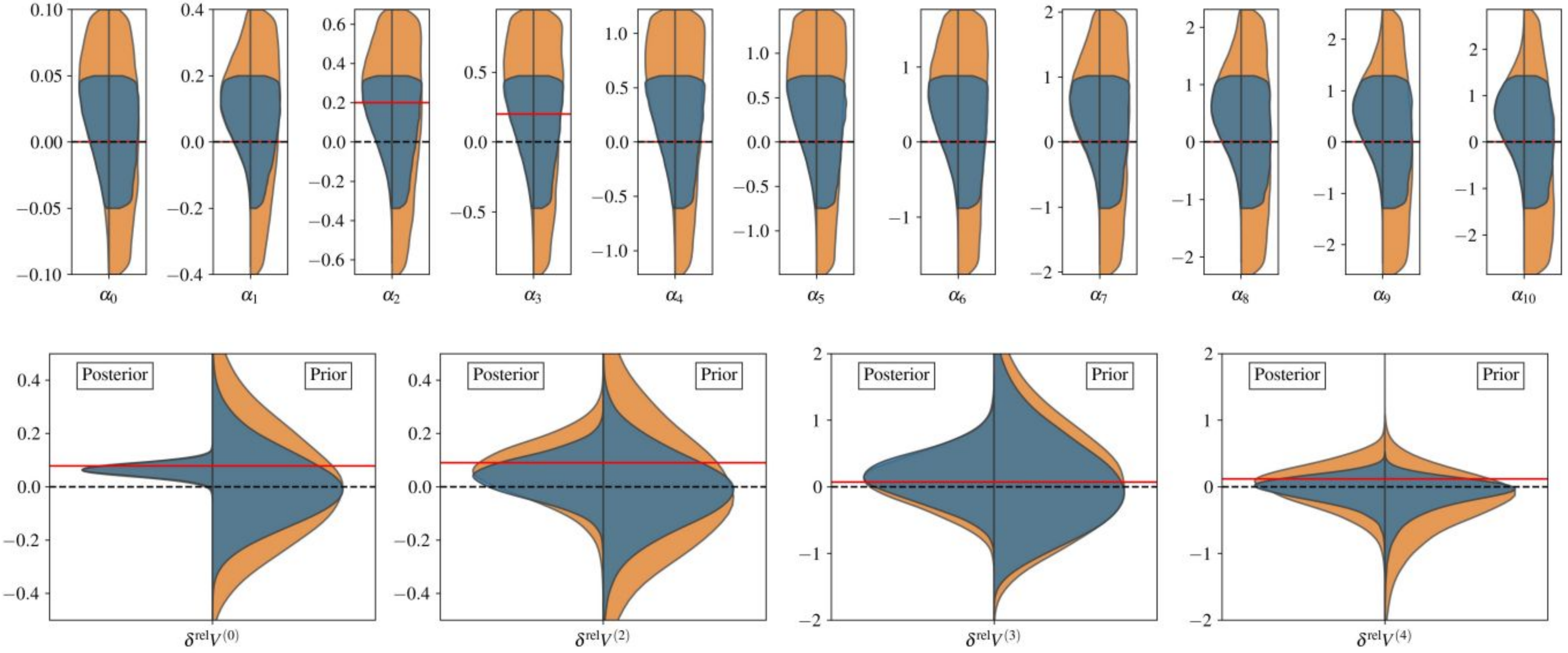


Volkel, NF, Barausse 2022
Volkel, NF, Barausse, Berti 2022

Agnostic effective potential

Parametrized ringdown formalism - nonrotating case

MCMC single parameters vs WKB: beyond-GR injection (red line)



Agnostic effective potential

Parametrized ringdown formalism - rotating case

Radial Teukolsky equation

$$\frac{1}{\Delta^s R(r)}\frac{d}{dr}\left[\Delta^{s+1}R'(r)\right]+V(r)=0$$

where

$$V(r)=2\mathrm{i}s\frac{\mathrm{d}K}{\mathrm{d}r}-\lambda_{\ell m}+\frac{1}{\Delta}\left(K^2-\mathrm{i}sK\frac{\mathrm{d}\Delta}{\mathrm{d}r}\right)$$

$$\Delta=r^2-r+a^2\,,\qquad K=(r^2+a^2)\omega-am\,,$$

$$\lambda_{\ell m}=B_{\ell m}+a^2\omega^2-2am\omega\,.$$

Cano, Capuano, NF, Maenaut, Volkel *2024-A*

Agnostic effective potential

Parametrized ringdown formalism - rotating case

Modifications to the radial equation

$$\frac{1}{\Delta^s R(r)}\frac{d}{dr}\left[\Delta^{s+1}R'(r)\right]+V(r)+\delta V(r)=0$$

corrections depending on the theory

where

$$V(r)=2\mathrm{i}s\frac{\mathrm{d}K}{\mathrm{d}r}-\lambda_{\ell m}+\frac{1}{\Delta}\left(K^2-\mathrm{i}sK\frac{\mathrm{d}\Delta}{\mathrm{d}r}\right)$$

$$\Delta=r^2-r+a^2\,,\qquad K=(r^2+a^2)\omega-am\,,$$

$$\lambda_{\ell m}=B_{\ell m}+a^2\omega^2-2am\omega\,.$$

Cano, Capuano, NF, Maenaut, Volkel *2024-A*

Agnostic effective potential

Parametrized ringdown formalism - rotating case

$$\frac{1}{\Delta^s R(r)}\frac{d}{dr}\left[\Delta^{s+1}R'(r)\right] + V(r) + \delta V(r) = 0$$

We take agnostic modification

$$\delta V(r) = \frac{1}{\Delta}\sum_{k=-K}^{4} \alpha^{(k)} \left(\frac{r}{r_+}\right)^k$$

Agnostic effective potential

Parametrized ringdown formalism - rotating case

$$\frac{1}{\Delta^s R(r)}\frac{d}{dr}\left[\Delta^{s+1}R'(r)\right]+V(r)+\delta V(r)=0$$

We take agnostic modification

$$\delta V(r)=\frac{1}{\Delta}\sum_{k=-K}^{4}\alpha^{(k)}\left(\frac{r}{r_+}\right)^k$$

All these coefficients assumed proportional to $\zeta \ll 1$

Cano, Capuano, NF, Maenaut, Volkel *2024-A*

Agnostic effective potential

Parametrized ringdown formalism - rotating case

We take agnostic modification leading to QNM shifts

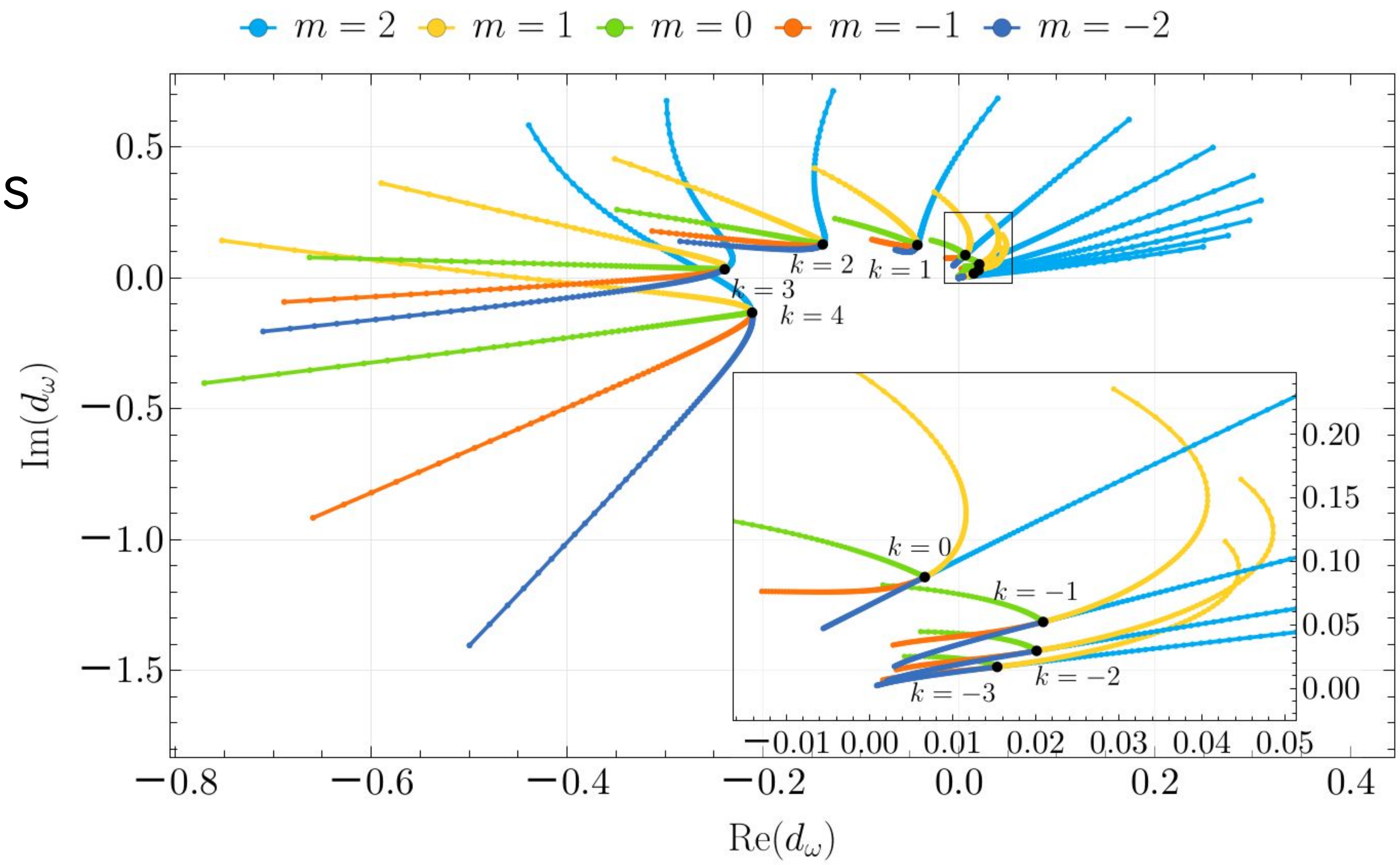
$$\omega_{n\ell m} \simeq \omega^{0}_{n\ell m} + \sum_k d^{(k)}_{\omega,n\ell m}\alpha^{(k)}$$

universal coefficients

Agnostic effective potential

Parametrized ringdown formalism - rotating case

Frequency coefficients
for *n=0, l=2*



Available with
tutorial on [github]

Cano, Capuano, NF, Maenaut, Volkel *2024-A*
Cano, Capuano, NF, Maenaut, Volkel *2024-B*

Parametrized ringdown formalism

Explicit case: Higher Derivative Gravity

$$S_{\mathrm{HDG}} = \frac{1}{16\pi}\int d^4x\sqrt{g}\left[R+\lambda_{\mathrm{ev}} R_{ab}{}^{cd} R_{cd}{}^{ef} R_{ef}{}^{ab}\right]$$

Manipulation of the equations to get

$$\delta\omega^{\pm} = \frac{\omega^{\pm}-\omega^{\mathrm{KERR}}}{\lambda} = \sum_{k\in k^{\mathrm{HD}}} A_{\pm}^{(k)} r_{+}^{k} d_{(k)}$$

$$k^{\mathrm{HD}} = [-2, 0, 1, 2]$$

Up to 18th order in the spin + Pade spin resummation

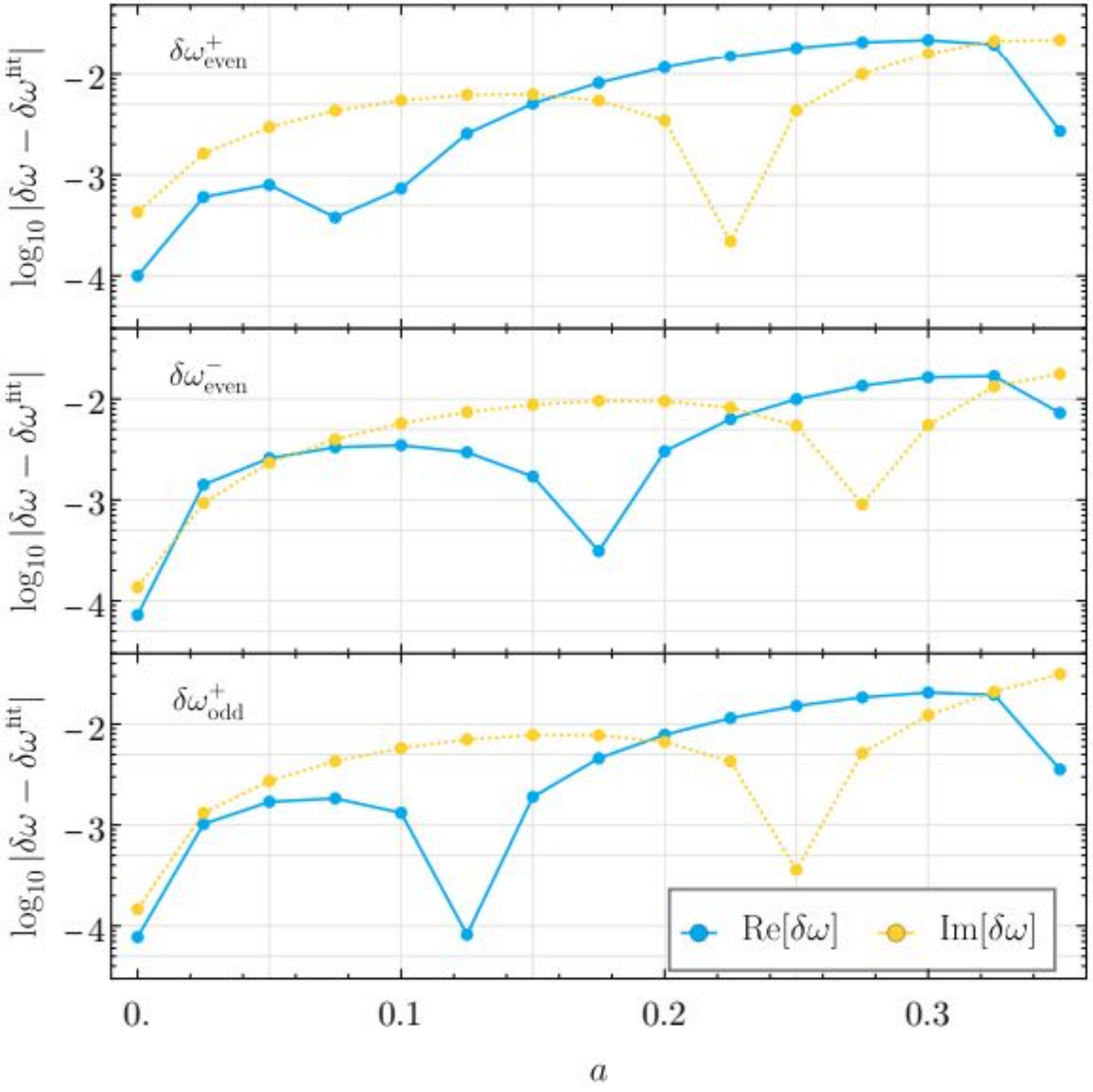
Parametrized ringdown formalism

Explicit case: Higher Derivative Gravity

Cano, Capuano, NF, Maenaut, Volkel *2024-A*
Cano, Capuano, NF, Maenaut, Volkel *2024-B*

Comparison of fits [Cano, Fransen, Hertog, Maenaut 2023]

Polar (even, odd) and axial (even) *n=0, l=2*



(Units M = 1/2, gives a=[0, 0.5])

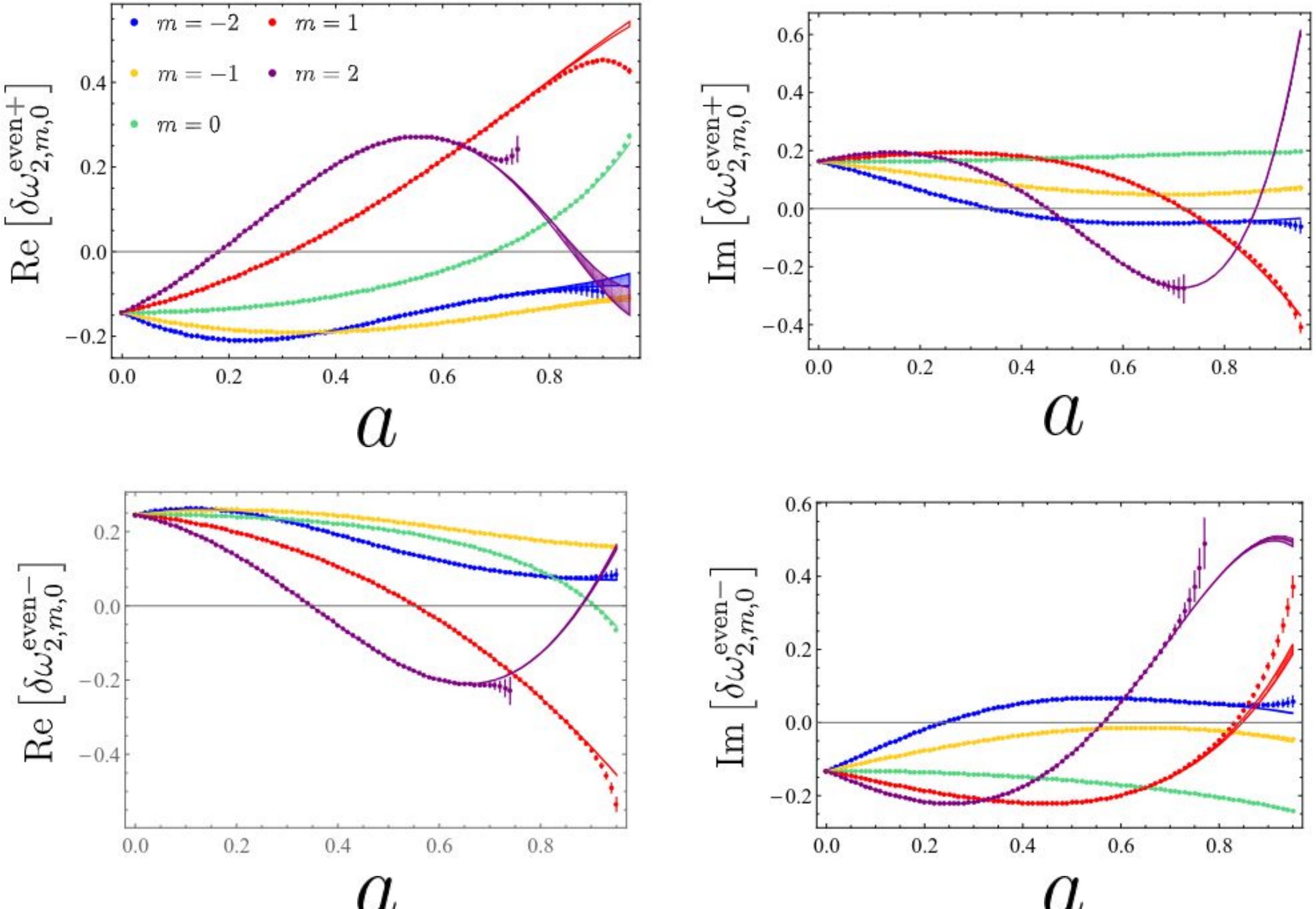
Cano, Capuano, NF, Maenaut, Volkel *2024-A*
Cano, Capuano, NF, Maenaut, Volkel *2024-B*

Parametrized ringdown formalism

Explicit case: Higher Derivative Gravity

First accurate computation of beyond-GR QNMs

- *n=[0,2] l=[2,4]*
- up to *a=[0.7,0.9]*
- Ready for tests against real data



Agnostic effective potential

Parametrized ringdown formalism - rotating WKB case

Is it possible to perform WKB with rotation?

PHYSICAL REVIEW D VOLUME 41, NUMBER 2 15 JANUARY 1990

Black-hole normal modes: A WKB approach. IV. Kerr black holes

Edward Seidel* and Sai Iyer†

McDonnell Center for the Space Sciences, Department of Physics, Washington University, St. Louis, Missouri 63130

(Received 24 July 1989)

Using the higher-order WKB method developed by Iyer and Will, we have computed the low-lying normal modes of Kerr black holes for both scalar and gravitational perturbations. For the gravitational modes, we compare our results to previously published numerical results. For some of these modes, we find agreement to within 1% for both the real and imaginary parts of the normal-mode frequency over a wide range of values for the rotation parameter *a* of the black hole. For other modes, good agreement is limited to lower values of *a*. The difficulties of the method for higher values of the rotation parameter are discussed.

Normal modes of the Kerr black hole

K D Kokkotas

Department of Physics, Section Astrophysics, Astronomy and Mechanics, University of Thessaloniki, Thessaloniki 54006, Greece

Received 2 May 1990, in final form 21 May 1991

Abstract. A new way to derive the Schutz–Will formula for the calculation of the quasi normal mode (QNM) frequencies is presented. The method is based on the Bohr–Sommerfeld rule and it is applicable even for the frequencies with large damping. Using this method the normal mode frequencies of the Kerr black hole have been calculated.

Agnostic effective potential

Parametrized ringdown formalism - rotating WKB case

$$Q_N \equiv \left. \frac{\mathrm{d}^N Q}{\mathrm{d}r_*^N} \right|_{\bar{r}_*}$$

Derivatives of the potential at the peak

Is it possible to perform WKB with rotation?

Simultaneous solution of

$$Q_0 + \mathrm{i}\beta\sqrt{2Q_2} = 0$$
$$Q_1(\omega, \bar{r}_*) = 0$$

- The potential il complex (bi-dimensional maximum)
- The solution is numerical
- Precision degrades with spin

Seidel, Iyer 1989
Kokkotas 1990

Tang, NF, Volkel, Berti *2025*

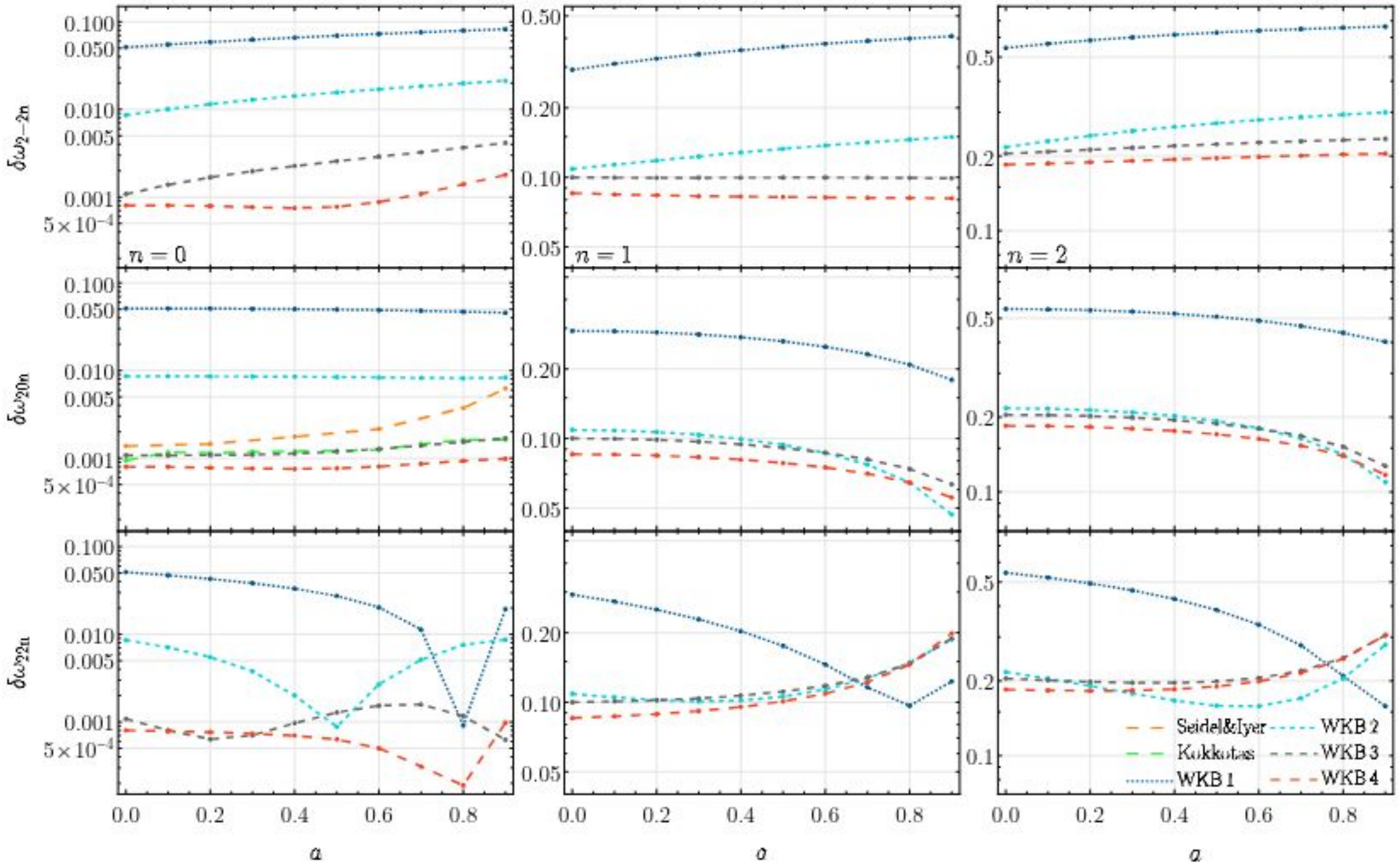
Agnostic effective potential

Parametrized ringdown formalism - rotating WKB case

Kerr QNMs

Comparison between Leaver and WKB

- *n=[0,2] l=[2,4]*
- up to *a=0.9*
- WKB order = [1,4]



Tang, NF, Volkel, Berti *2025*

Agnostic effective potential

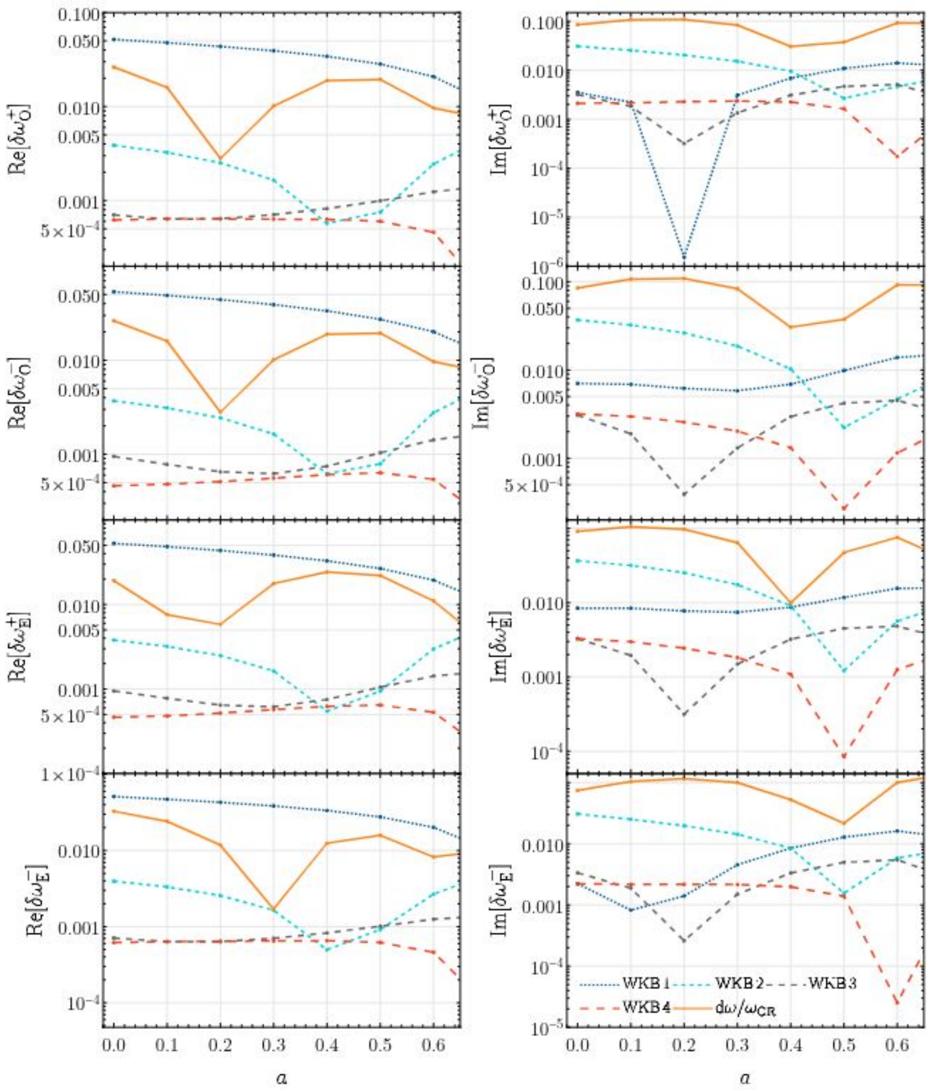
Parametrized ringdown formalism - rotating WKB case

Higher derivative gravity QNMs

Comparison between Leaver and WKB

- *n=[0,2] l=[2,4]*
- up to *a=0.7*
- WKB order = [1,4]

Difference order 0.1 ~ 0.5%



Agnostic parametrizations

- Mass-spin parametrization (ParSpec):
  - flexible and general
  - too many parameters

- Non-rotating potential parametrization:
  - small coupling assumption
  - reduction of parameters with PCA or WKB-inspired expansion

- Rotating potential parametrization:
  - small coupling assumption
  - too few cases for calibration (work in progress)
  - WKB-inspired expansion possible (work in progress)

Can we improve ringdown tests of GR?

Overall takeaway:

- Use of all ringdown quantities
- Tests independent from IMR analysis
- Parametrizations that depend on a few parameters
- Connect parameters to dynamics and metric of deviations
- Clear mapping to known theories

Work in progress towards a unified prescription of deviations from GR in the ringdown

Can we improve ringdown tests of GR?

Overall takeaway:

- Use of all ringdown quantities
- Tests independent from IMR analysis
- Parametrizations that depend on a few parameters
- Connect parameters to dynamics and metric of deviations
- Clear mapping to known theories

Work in progress towards a unified prescription of deviations from GR in the ringdown

Thanks for the attention!