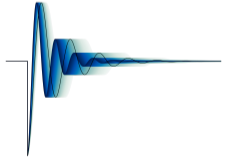


Finding the ringdown of rotating black holes in higher-derivative gravity



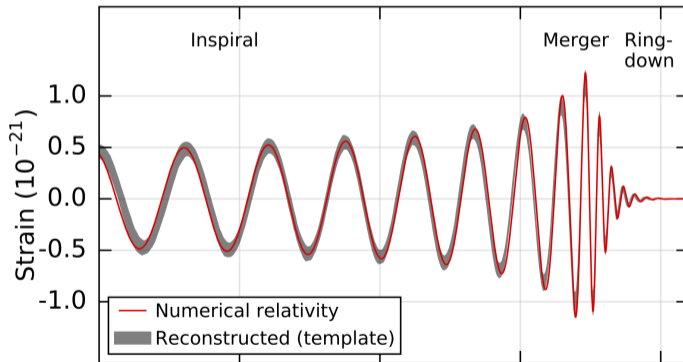
Simon Maenaut

5 June 2025



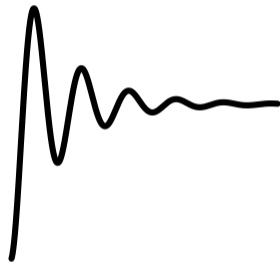
[2304.02663, 2307.07431] with Pablo Cano, Kwinten Fransen and Thomas Hertog
[2407.15947, 2409.04517] with Pablo Cano, Lodovico Capuano, Nicola Franchini and Sebastian Völkel
[2411.17893] with Pablo Cano, Vitor Cardoso, Gregorio Carullo, Thomas Hertog, Tjonnie Li and Anna Liu

Gravitational Wave from a Black Hole Merger



Ringdown Phase

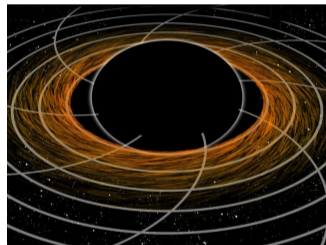
- Perturbations of black hole decay over time
- GWs come from space around the black hole
- Fluctuations of a damped harmonic oscillator
- Boundary conditions set a dissipative system
- Resonance modes have complex frequencies



$$\sum A_k \exp(-i\omega_k t)$$

Quasinormal Modes in General Relativity

- In General Relativity (4 dim.) black holes are special
- These are fully defined by their mass, spin and charge*
- A black hole has no additional characteristics (no hair)
- Characteristic discrete spectrum of resonance modes
- Check observations with quasinormal modes of Kerr



Newman-Penrose Formalism

- Introduce a tetrad frame set by four null vectors $l^\mu, n^\mu, m^\mu, \bar{m}^\mu$
- The vectors l^μ and n^μ are in- and outgoing null directions
- The other null vectors have to be complex m^μ and \bar{m}^μ
- Since the metric is real, these are complex conjugates
- The tangent space metric is fixed to a specific form

$$g_{\mu\nu} = -l_\mu n_\nu - n_\mu l_\nu + m_\mu \bar{m}_\nu + \bar{m}_\mu m_\nu$$

Weyl Scalars

- Traceless part of the Riemann tensor, is the Weyl tensor $C_{\alpha\beta\mu\nu}$
- Can be written in terms of 5 independent complex components
- Invariant under diffeomorphisms, dependant on the tetrad basis

$$\begin{aligned}\Psi_0 &= C_{\alpha\beta\mu\nu} l^\alpha m^\beta l^\mu m^\nu & \Psi_1 &= C_{\alpha\beta\mu\nu} l^\alpha n^\beta l^\mu m^\nu & \Psi_2 &= C_{\alpha\beta\mu\nu} l^\alpha m^\beta \bar{m}^\mu n^\nu \\ \Psi_3 &= C_{\alpha\beta\mu\nu} l^\alpha n^\beta \bar{m}^\mu n^\nu & \Psi_4 &= C_{\alpha\beta\mu\nu} n^\alpha \bar{m}^\beta n^\mu \bar{m}^\nu\end{aligned}$$

Teukolsky Equations

- Kerr metric is special (Petrov type D), especially with the Kinnersley tetrad
- Its only non-vanishing Weyl scalar is $\Psi_2 = -M/\zeta$ with $\zeta = r - ia \cos \theta$
- Perturbations can be solved in terms of $\delta\Psi_0$ or $\delta\Psi_4$ equivalently
- The equations for these two perturbed Weyl scalars are separable
- Physically, $\delta\Psi_0$ and $\delta\Psi_4$ describe the two polarization modes of GWs

$$\psi = R_{lm}^s(r) S_{lm}^s(\cos \theta) \exp(-i\omega t + im\phi)$$

Effective Field Theory

- Consider General Relativity in an effective field theory of gravity
- A good approximation to study the phenomena at one energy scale
- Corrections are possible from the full theory at a higher energy scales

$$S = \frac{1}{16\pi} \int d^4x \sqrt{|g|} \left[R + \sum_{n=2}^{\infty} \ell^{2n-2} \mathcal{L}_{(n)} \right]$$

Higher-derivative Gravity

$$S = \frac{1}{16\pi} \int d^4x \sqrt{|g|} \left[R + \ell^4 \left(\lambda_{\text{ev}} R^3 + \lambda_{\text{odd}} \tilde{R}^3 \right) + \mathcal{O}(\ell^6) \right]$$

$$R^3 = R_{\mu\nu}{}^{\rho\sigma} R_{\rho\sigma}{}^{\delta\gamma} R_{\delta\gamma}{}^{\mu\nu}, \quad \tilde{R}^3 = R_{\mu\nu}{}^{\rho\sigma} R_{\rho\sigma}{}^{\delta\gamma} \tilde{R}_{\delta\gamma}{}^{\mu\nu}$$

where $\tilde{R} = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} R^{\alpha\beta}{}_{\rho\sigma}$ and ℓ is a length scale related to the cutoff of the EFT.

For a black hole of mass M , we introduce the following dimensionless couplings: $\alpha_{\text{ev}} = \lambda_{\text{ev}} \ell^4 / M^4$ and $\alpha_{\text{odd}} = \lambda_{\text{odd}} \ell^4 / M^4$, which are assumed to be very small.

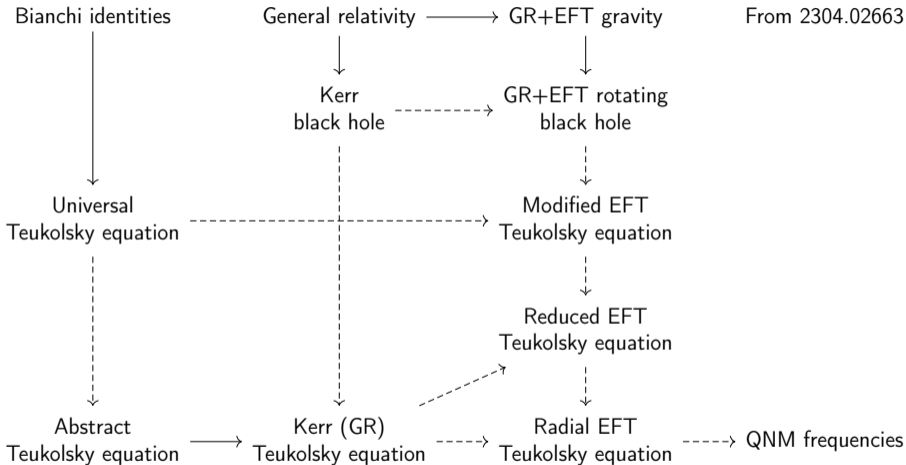
Corrected Kerr Metric

$$ds^2 = - \left(1 - \frac{2Mr}{\Sigma} - H_1 \right) dt^2 - (1 + H_2) \frac{4aMr \sin^2 \theta}{\Sigma} dt d\phi$$
$$+ (1 + H_3) \Sigma \left(\frac{dr^2}{\Delta} + d\theta^2 \right) + (1 + H_4) \left(r^2 + a^2 + \frac{2a^2 Mr \sin^2 \theta}{\Sigma} \right) \sin^2 \theta d\phi^2$$

where $\Sigma = r^2 + a^2 \cos^2 \theta$ and $\Delta = r^2 - 2Mr + a^2$, while the additional functions $H_j(r, \theta)$ describe the first order corrections to the Kerr metric in BL coordinates.

Theoretical Framework

- Study the the effect of higher-derivative corrections on quasinormal modes
- The metric background and also the dynamics of perturbations are modified
- Formulate combination of Bianchi identities that reduce to Teukolsky equation
- Universal Teukolsky equations apply to any background geometry in any theory
- The theory dependence only enters through the effective stress-energy tensor



Universal Teukolsky Equation

The schematic form of the Teukolsky equation is $\hat{D}[e_a^\mu, \gamma_{abc}] \left(\nabla_{[\gamma} R_{\alpha\beta]\sigma\rho} \right) = 0$ and is a linear differential operator acting on the differential Bianchi identities.

Follow the Newman-Penrose (NP) and Geroch-Held-Penrose (GHP) approaches, however, without making any assumptions on the spacetime metric or NP-frame.

$$\begin{aligned} \mathcal{O}_2^{(0)}(\Psi_0) + \mathcal{O}_2^{(1)}(\Psi_1) + \mathcal{O}_2^{(2)}(\Psi_0) &= 8\pi \left(\mathcal{T}_2^{(0)} + \mathcal{T}_2^{(1)} + \mathcal{T}_2^{(2)} \right) \\ \mathcal{O}_{-2}^{(0)}(\Psi_4) + \mathcal{O}_{-2}^{(1)}(\Psi_3) + \mathcal{O}_{-2}^{(2)}(\Psi_4) &= 8\pi \left(\mathcal{T}_{-2}^{(0)} + \mathcal{T}_{-2}^{(1)} + \mathcal{T}_{-2}^{(2)} \right) \end{aligned}$$

Perturbations

- These equations involve e_a , γ_{abc} , ϕ_{ab} and Ψ_i and take the form $\mathcal{E}_{\pm 2}(\Phi, \Psi_i) = 0$
- Consider a perturbation over a solution, then $\Psi \rightarrow \bar{\Psi} + \delta\Psi$ and $\Phi \rightarrow \bar{\Phi} + \delta\Phi$
- Assume we are close to the Teukolsky equations to use their nice properties

$$\mathcal{E}_{\delta\Psi, \pm 2}(\delta\Psi_i) + \mathcal{E}_{\delta\Phi, \pm 2}(\delta\Phi) = 0$$

Linearisation

Introduce λ to keep track of the small departure from vacuum Petrov type D

- The background geometry has $\bar{\Psi}_i = \bar{\Psi}_i^{(0)} + \lambda \bar{\Psi}_i^{(1)}$ and $\bar{\Phi}_i = \bar{\Phi}_i^{(0)} + \lambda \bar{\Phi}_i^{(1)}$
- The operators $\mathcal{E}_{\delta\Phi, \pm 2}$ become of order λ , so $\mathcal{E}_{\delta\Phi, \pm 2}(\delta\Phi) = \lambda \mathcal{E}_{\delta\Phi, \pm 2}^{(1)}(\delta\Phi)$
- While $\mathcal{E}_{\delta\Psi, \pm 2}(\delta\Psi_i) = \mathcal{D}_{\pm 2}^{(0)}(\delta\Psi_{0,4}) + \lambda \mathcal{E}_{\delta\Psi, \pm 2}^{(1)}(\delta\Psi_i)$ for the Weyl scalars

$$\begin{aligned}\mathcal{D}_{+2}^{(0)}(\delta\Psi_0) + \lambda \left[\mathcal{E}_{\delta\Psi, +2}^{(1)}(\delta\Psi_i) + \mathcal{E}_{\delta\Phi, +2}^{(1)}(\delta\Phi) \right] &= 0, \\ \mathcal{D}_{-2}^{(0)}(\delta\Psi_4) + \lambda \left[\mathcal{E}_{\delta\Psi, -2}^{(1)}(\delta\Psi_i) + \mathcal{E}_{\delta\Phi, -2}^{(1)}(\delta\Phi) \right] &= 0.\end{aligned}$$

Reduction

Every zeroth-order perturbed quantity is written in terms of $\delta\Psi_0^{(0)}$ and $\delta\Psi_4^{(0)}$

$$\delta\Phi^{(0)} = \delta\Phi^{(0)}(\delta\Psi_0^{(0)}, \delta\Psi_4^{(0)}) = \delta\Phi^{(0)}(\delta\Psi_0, \delta\Psi_4) + \mathcal{O}(\lambda).$$

to form two coupled equations for the two variables $\delta\Psi_{0,4}$ (including corrections).

$$\begin{aligned} \mathcal{D}_{+2}^{(0)}(\delta\Psi_0) + \lambda \left[\mathcal{E}_{\delta\Psi_0,+2}^{(1)}(\delta\Psi_0) + \mathcal{E}_{\delta\Psi_4,+2}^{(1)}(\delta\Psi_4) \right] &= 0, \\ \mathcal{D}_{-2}^{(0)}(\delta\Psi_4) + \lambda \left[\mathcal{E}_{\delta\Psi_0,-2}^{(1)}(\delta\Psi_0) + \mathcal{E}_{\delta\Psi_4,-2}^{(1)}(\delta\Psi_4) \right] &= 0. \end{aligned}$$

Considerations

To carry out this computation we need two ingredients:

1. the Newman-Penrose description of the background geometry

(including higher-derivative corrections) $e_{(1)a}{}^{\mu} = -\frac{1}{2}\bar{g}_{\alpha\beta}^{(1)}\bar{g}^{(0)\mu\alpha}e_{(0)a}{}^{\beta}$,

2. the solution for every perturbed quantity in General Relativity

(so without corrections) expressed in terms of $\delta\Psi_0$ and $\delta\Psi_4$.

For complex metric perturbations, the conjugates in the NP formalism are not actual complex conjugates. Instead, the conjugate variables become independent.

Metric Reconstruction

We can reconstruct the metric perturbation on the Kerr background

$$M\partial_t h_{\mu\nu} = -\frac{1}{3}\nabla_\beta \left[\zeta^4 \nabla_\alpha C_{(\mu \nu)}^{\alpha \beta} \right] - \frac{1}{3}\nabla_\beta \left[(\zeta^*)^4 \nabla_\alpha \bar{C}_{(\mu \nu)}^{\alpha \beta} \right]$$

$$C_{\mu\alpha\nu\beta} = 4 \left(\psi_0 n_{[\mu} \bar{m}_\alpha] n_{[\nu} \bar{m}_\beta] } + \psi_4 l_{[\mu} m_\alpha] l_{[\nu} m_\beta] } \right) ,$$

$$\bar{C}_{\mu\alpha\nu\beta} = 4 \left(\psi_0^* n_{[\mu} m_\alpha] n_{[\nu} m_\beta] } + \psi_4^* l_{[\mu} \bar{m}_\alpha] l_{[\nu} \bar{m}_\beta] } \right) .$$

From $h_{\mu\nu}$ we find the perturbation of the NP frame $\delta e_a^\mu = -\frac{1}{2} h_{\alpha\beta} \bar{g}^{\mu\alpha} \bar{e}_a^\beta$.

Teukolsky Variables

The variables $\psi_{0,4}$ and $\psi_{0,4}^*$ satisfy the usual Teukolsky equations, however, they are in general only proportional to the Weyl scalars $\delta\Psi_{0,4}$ and $\delta\Psi_{0,4}^*$.

Since these variables satisfy Teukolsky equations, they can be separated as

$$\begin{aligned}\delta\Psi_0 &= P_{+2}\psi_0 = e^{-i\omega t + im\phi} P_{+2}R_{+2}(r)S_{+2}(x), \\ \delta\Psi_4 &= P_{-2}\psi_4 = e^{-i\omega t + im\phi} \zeta^{-4} P_{-2}R_{-2}(r)S_{-2}(x), \\ \delta\Psi_0^* &= P_{+2}^*\psi_0^* = e^{-i\omega t + im\phi} P_{+2}^*R_{+2}^*(r)S_{-2}(x), \\ \delta\Psi_4^* &= P_{-2}^*\psi_4^* = e^{-i\omega t + im\phi} (\zeta^*)^{-4} P_{-2}^*R_{-2}^*(r)S_{+2}(x).\end{aligned}$$

Starobinsky-Teukolsk identities

$$\begin{aligned}R_{+2}^*(r) &= q_{+2}R_{+2}(r), & R_{-2} &= C_{+2}\Delta^2 (\mathcal{D}_0)^4 (\Delta^2 R_{+2}), \\R_{-2}^*(r) &= q_{-2}R_{-2}(r), & R_{+2} &= C_{-2} (\mathcal{D}_0^\dagger)^4 R_{-2},\end{aligned}$$

$$\begin{aligned}\mathcal{D}_0 &= \partial_r + i(\omega(r^2 + a^2) - ma)/\Delta, & C_{+2}C_{-2} &= 1/\mathcal{K}^2, \\ \mathcal{D}_0^\dagger &= \partial_r - i(\omega(r^2 + a^2) - ma)/\Delta, & \mathcal{K}^2 &= D_2^2 + 144M^2\omega^2.\end{aligned}$$

Polarization parameters $q_{\pm 2}$ and proportionality constants $C_{\pm 2}$ set $P_{\pm 2}$ and $P_{\pm 2}^*$.

Projection and Separation

Spin-weighted spheroidal harmonics satisfy the orthogonality relations:

$$2\pi \int_{-1}^1 dx S_s^{lm}(x; a\omega) S_s^{l'm}(x; a\omega) = \delta_{ll'}$$

We can project the equations onto $S_s^{l'm}$ and obtain an infinite system of radial equations labelled by the number l' . However since for Kerr there is a single term, with a fixed l and m , we have one leading term while the rest comes in at order λ .

Radial Master Equation

With the relations at zeroth order in λ , from the Teukolsky equations for Kerr:

$$-P_s \mathfrak{D}_s^2 R_s + \lambda \left[f_{s,0} R_s + f_{s,1} \frac{dR_s}{dr} \right] = 0, \quad -P_s^* \mathfrak{D}_s^2 R_s^* + \lambda \left[f_{s,0}^* R_s^* + f_{s,1}^* \frac{dR_s^*}{dr} \right] = 0$$

With a change of variables, we can write a radial master equation of the form:

$$\Delta^{-s+1} \frac{d}{dr} \left[\Delta^{s+1} \frac{dR_s}{dr} \right] + (V_s + \lambda \delta V_s) R_s = 0$$

Parametric approach

$$\Delta^{-s+1} \frac{d}{dr} \left[\Delta^{s+1} \frac{dR_s}{dr} \right] + (V_s + \lambda \delta V_s) R_s = 0$$

Consider a generic modification to the Teukolsky equation

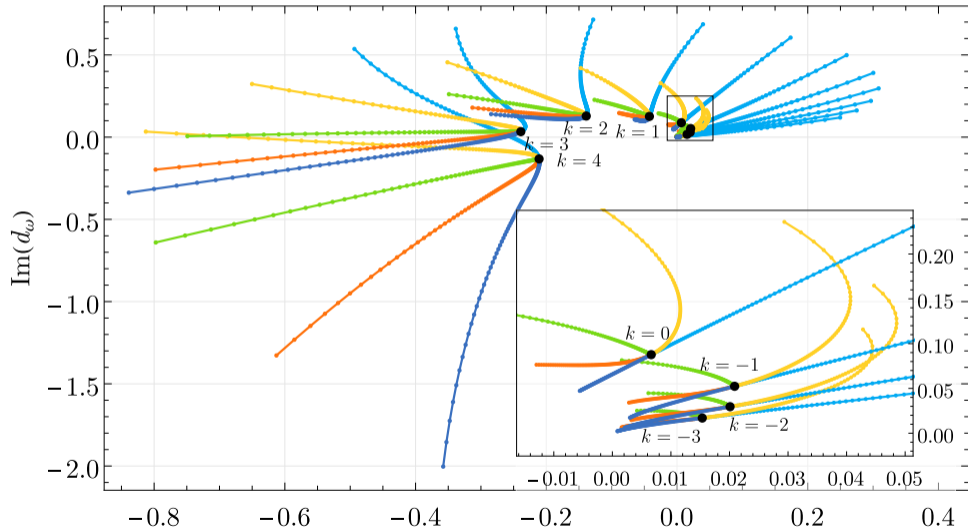
$$\delta V_s = \sum_{k=-K}^4 \alpha^{(k)} \left(\frac{r}{r_+} \right)^k$$

Frequency shifts

These lead to a shift in the frequencies and separation constant

$$\omega_{lmn} = \omega_{lmn}^{\text{Kerr}} + \sum_k d_{\omega,lmn}^{(k)} \alpha^{(k)}$$
$$B_{lmn} = B_{lm}^{\text{Kerr}} + \sum_k d_{B,lmn}^{(k)} \alpha^{(k)}$$

● $m = 2$ ● $m = 1$ ● $m = 0$ ● $m = -1$ ● $m = -2$



Higher-derivative corrections

Consider a change of variables

$$R_s \rightarrow R_s + \lambda \left(A_s R_s + B_s \Delta \frac{dR_s}{dr} \right)$$

The choice of A_s and B_s makes it possible to write

$$\Delta^{-s+1} \frac{d}{dr} \left[\Delta^{s+1} \frac{dR_s}{dr} \right] + (V_s + \delta V_s) R_s = 0 \quad \text{with} \quad \delta V_s = \sum_{n=-2}^2 A_n r^n.$$

Quasinormal Mode Frequencies

$$\omega_s = \omega_{\text{Kerr}} + \lambda \delta \omega_s \quad \text{and} \quad \omega_s^* = \omega_{\text{Kerr}}^* + \lambda \delta \omega_s^*$$

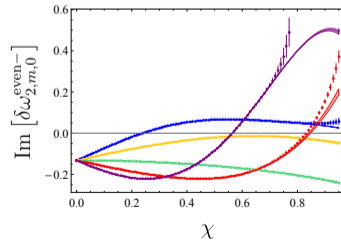
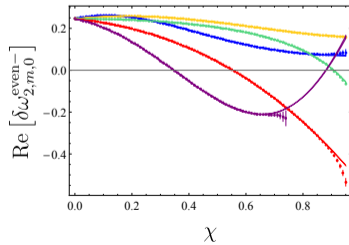
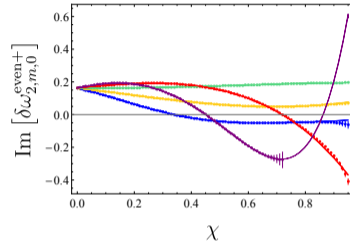
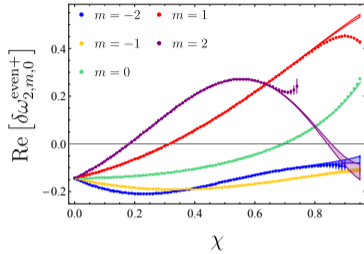
- All the corrections are in the effective master equation potential δV_s and δV_s^*
- We can calculate its first order effect on the complex frequency of each QNM
- For specific polarisation parameters $q_{\pm 2}$ all QNM frequency shifts should equal

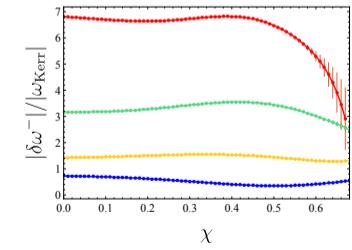
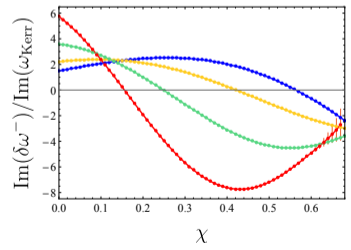
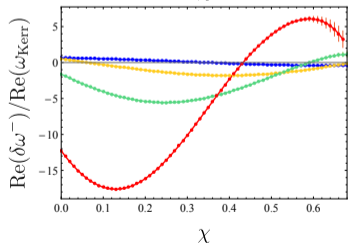
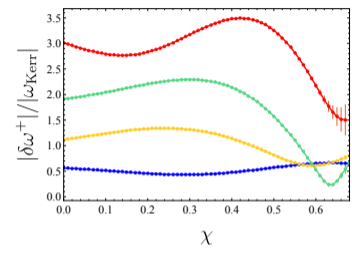
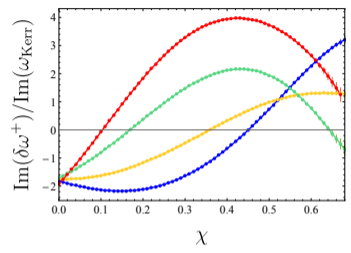
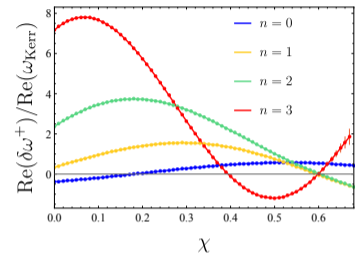
$$\delta \omega_{+2}(q_s, C_s) = \delta \omega_{-2}(q_s, C_s) = \delta \omega_{+2}^*(q_s, C_s) = \delta \omega_{-2}^*(q_s, C_s).$$

Polarisation constants

- The value of the polarisation constants depends specifically on the type of higher-derivative theory and whether the correction preserves parity symmetry
- In parity preserving theories, opposite parity modes decouple $q_{+2} = q_{-2} = \pm 1$
- In parity breaking theories they couple and $q_{-2} = (iK_1)/(1 \pm \sqrt{K_1^2 + 1})$

$$q_{+2} q_{-2} = \pm 1$$

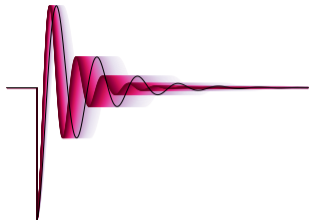




pyRing

Python package for black hole ringdown analysis

- Fully time domain formulation, both likelihood and waveform models
- Handles either observed, injected or simulated numerical relativity datasets
- Supports an extensive set of quasinormal mode spectra for different theories

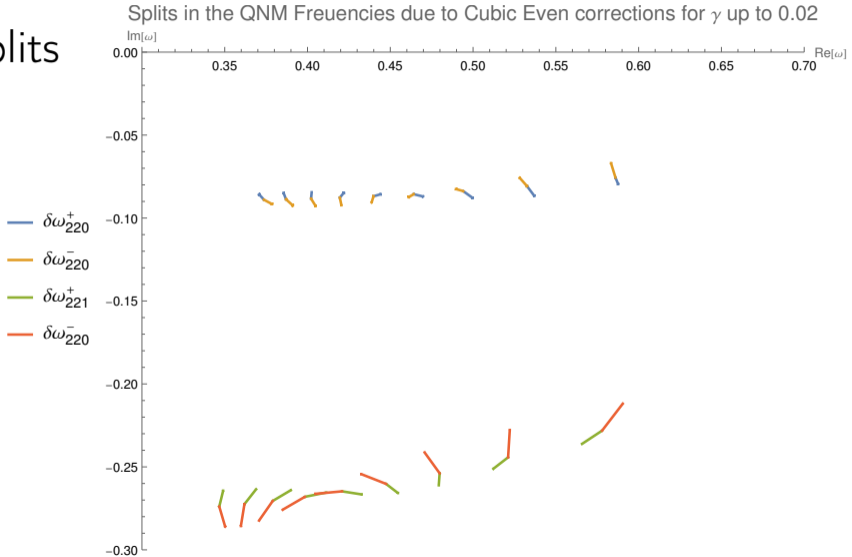


EFT Corrections

- Introduce a dimensionless correction parameter $\gamma_q = \lambda_q (\ell/M_{\text{src}})^p$
- Include redshift dependence of the observed mass $M_{\text{obs}} = M_{\text{src}} (1+z)$
- Set $\lambda_q = \pm 1$ and use the sign in ℓ such that $\gamma_q = \text{sign}(\ell) \left(\frac{c^2 |\ell| \cdot (1+z)}{G M_{\text{obs}}} \right)^p$

$$\omega^\pm = \omega_{\text{Kerr}} + \gamma_q \delta\omega_q^\pm, \quad \text{and} \quad \delta\omega_q^\pm(\chi) \approx \sum_{n=0}^N c_{q,n}^\pm \chi^n,$$

QNM Splits



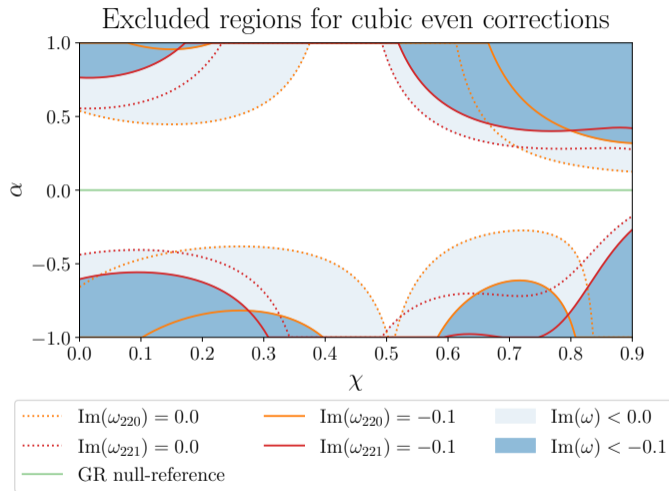
Assumptions

- Assume the EFT corrections are small $\gamma_q < 1$
- Require that the quasinormal mode is damped $\text{Im}[\omega] < 0$
- Apply the non-pressing symmetry for the amplitudes $A^{(P)} = (-1)^l A^{(N)}$

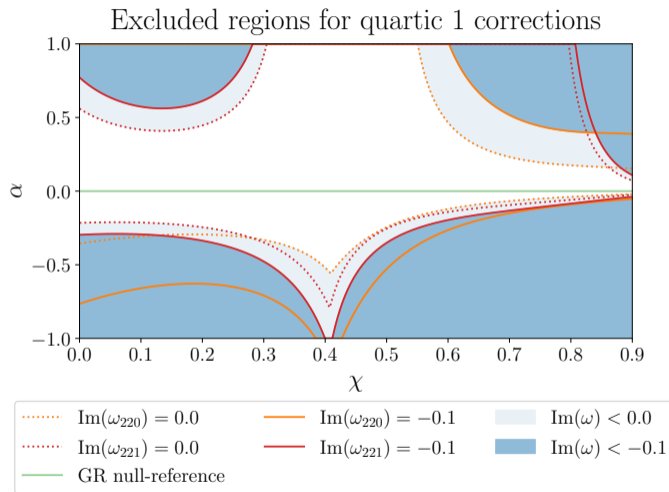
$$h_{lmn}^P = A^{(P)} S_{lmn}(i, \theta) e^{-i(\omega_{lmn}^\pm) t + i\phi_{lmn}}$$

$$h_{lmn}^N = A^{(N)} S_{lmn}(i, \theta) e^{+i(\omega_{lmn}^\pm)^* t + i\phi_{lmn}}$$

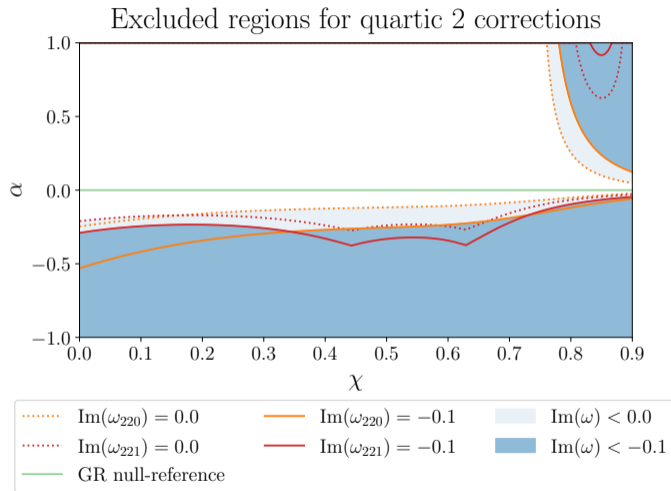
Limits



Limits

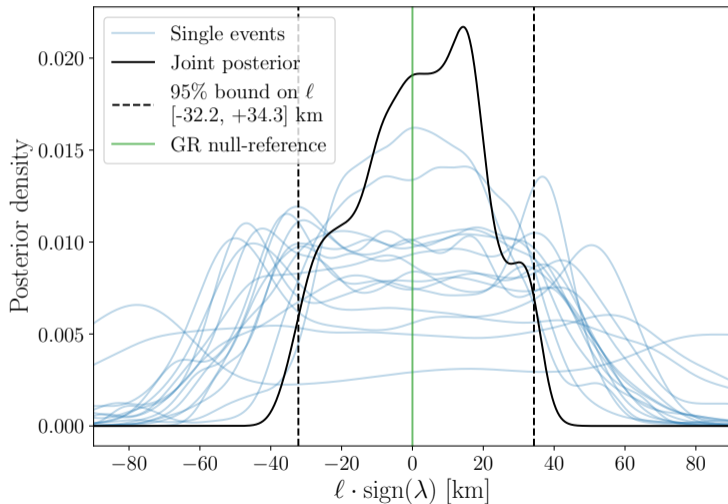


Limits



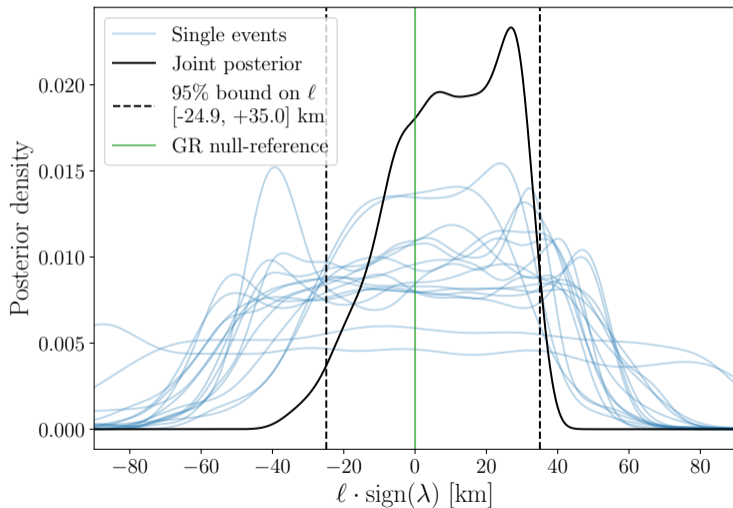
Results

Posterior distribution of ℓ for cubic even corrections

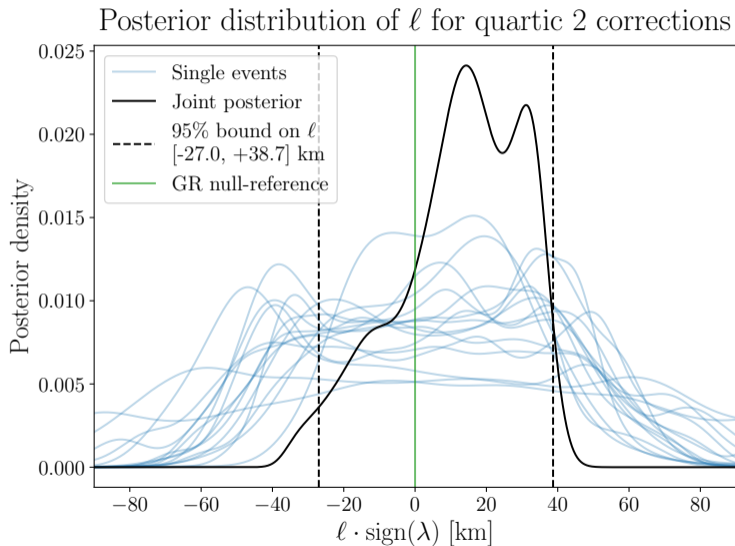


Results

Posterior distribution of ℓ for quartic 1 corrections



Results



Theory	$\text{sign}(\lambda) \cdot \ell$ in km	$\ln(\mathcal{B}_{\text{EFT}}/\mathcal{B}_{\text{GR}})$
Cubic even	$[-32.2, +34.3]$	$[-2.0, +1.0]$
Quartic 1	$[-24.9, +35.0]$	$[-2.1, +1.4]$
Quartic 2	$[-27.0, +38.7]$	$[-1.7, +0.9]$

Table: Summary of the constraints obtained at a 95 % level for each EFT correction, by combining posterior distributions of single events from GWTC-3, and the range of Bayes factors obtained from all events when comparing models.

Conclusion

- QNMs of rotating BHs with the universal Teukolsky equation
- Applied to the general EFT class of higher derivative gravity
- Correction coefficients were calculated up to ~ 0.9 in spin
- Implementation within pyRing using minimal assumptions
- Current GW observations set a constrain of $\ell < 35$ km

Some further ideas

- Investigate QNM frequency shifts near extremality
- Consider the effect of parity-breaking higher-derivative terms
- Include corrections with scalar fields (EdGB and dCS) in the framework
- Study a possible theory inspired parametrisation of QNM deviations
- Explore the sensitivity of future GW detectors to EFT corrections

Thank you for your attention!

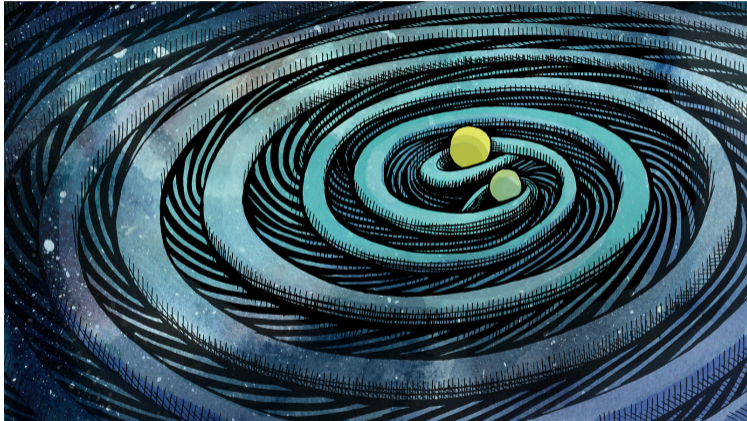
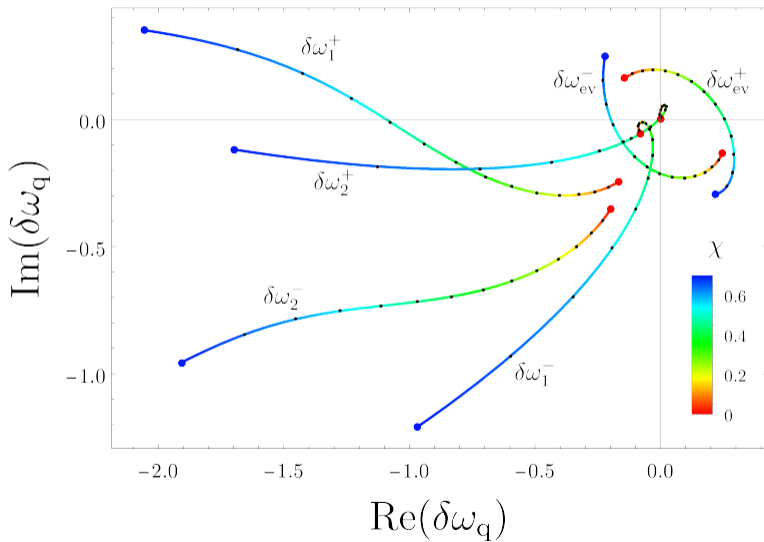
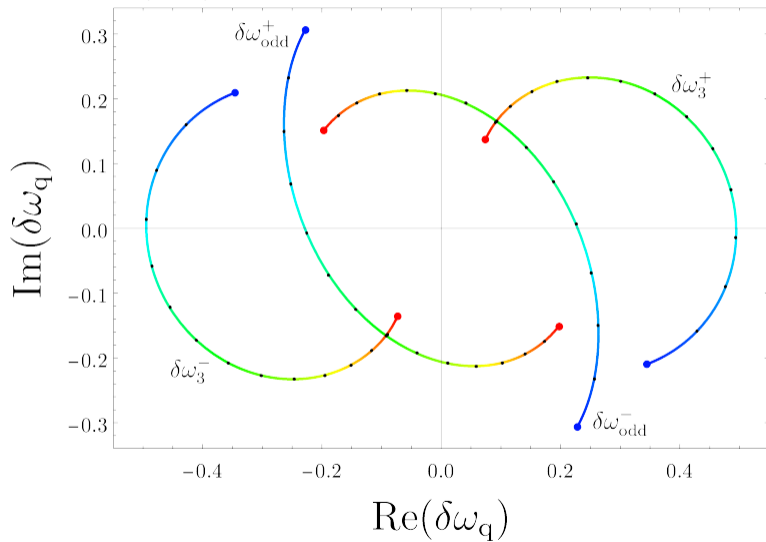


Figure: Illustration of the GWs emitted by a BH merger [Symmetry magazine]

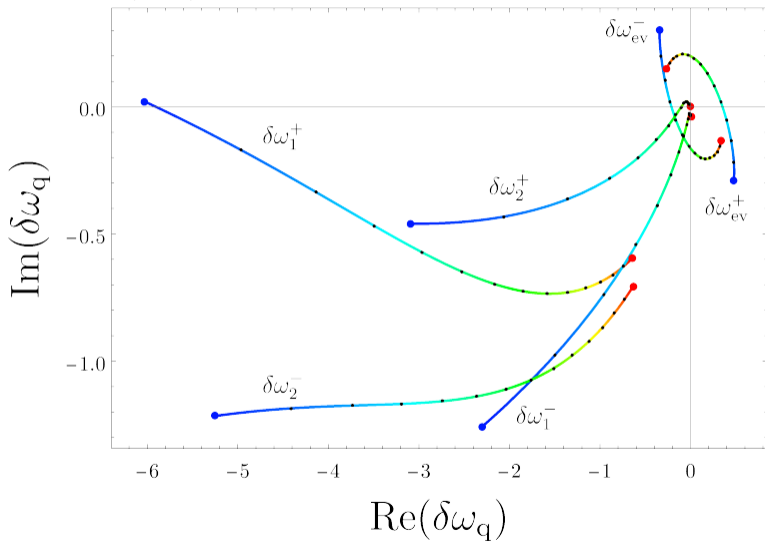
(220) modes: parity-preserving corrections



(220) modes: parity-breaking corrections



(330) modes: parity-preserving corrections



(330) modes: parity-breaking corrections

