

Simulation-Based-Inference (SBI) (for GWB reconstruction)

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Outline

- 1 Introduction
 - Measuring GWBs with LISA
 - Minimal working assumptions and their breakdown
- 2 SBI in a nutshell
 - General ideas
 - Neural Ratio Estimation (NRE) and Neural Posterior Estimation (NPE)
- 3 SBI for GWB and Saqqara
- 4 Some new ideas/algorithms for SBI
 - A convergence criterion for SBI
 - Dynamical (Round-free) Sequential SBI
- 5 Conclusions and outlook

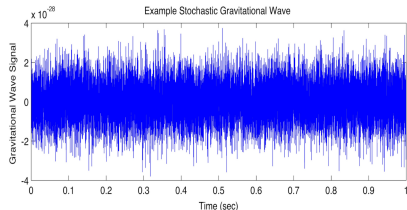
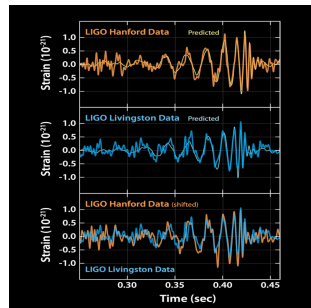
The dawn of GW astronomy

Gravitational Waves (GWs) are:

- Spacetime perturbations
- Almost free streaming
- The ultimate cosmological probe

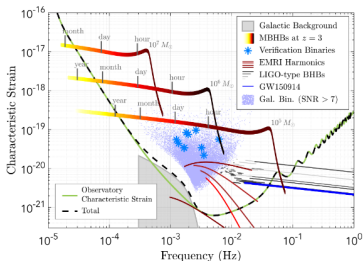
There are two classes of signals:

- **Deterministic signals**
 - Typically from binaries
 - Time coherency
 - Localized in the sky
- **GW Backgrounds (GWBs)**
 - Superposition of many signals
 - Early Universe processes
 - No time coherency
 - Diffuse signals



* Figures from <https://www.ligo.org/detections/images/ligoGW150914signals-lg.jpg>
and <https://www.ligo.org/science/GW-Overview/images/stochastic.jpg>

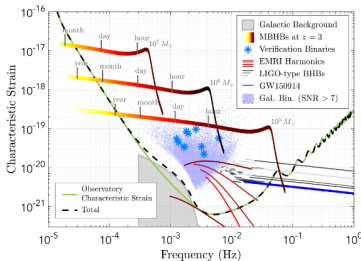
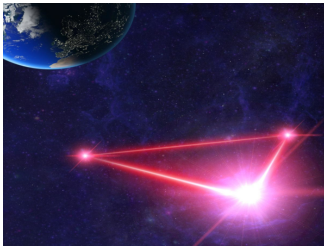
Laser Interferometer Space Antenna



Few details on **LISA**:

- First **GW** interferometer **in space**
- Constellation of three satellites
- **2.5 million km** arm lengths
- Peak sensitivity $10^{-2} \div 10^{-3}$ Hz
- Three correlated detectors
- Expected launch in mid **2030**
- Operating for **4.5 yrs (nominal)**

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Very interesting for cosmology since we can (among others):

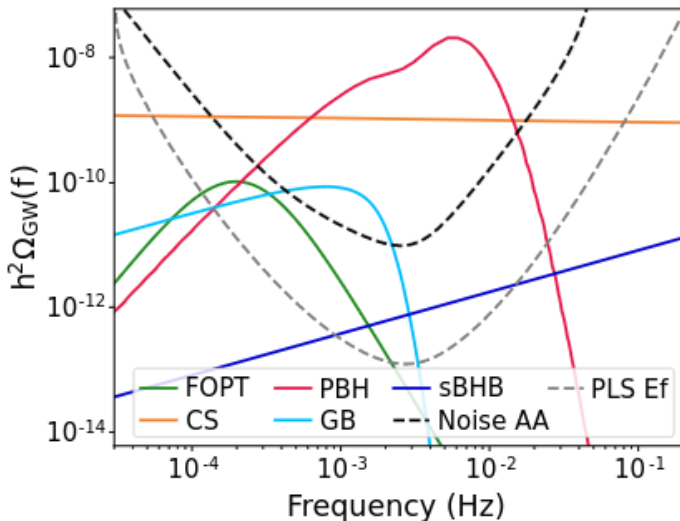
- Measure H_0
- Test modified gravity
- **(Hopefully) detect and characterize GWBs!**

* Figures from:

<https://www.lisamission.org/multimedia/image/lisa-astro2020>

LISA Collaboration, P. Amaro-Seoane et al., ArXiv: 1702.00786

Sources of GWBs in the LISA



* Figure from M. Colpi et al., ArXiv:2402.07571

Basics of GW data analysis

Data \tilde{d} (in frequency space) $\longrightarrow \tilde{d} = \tilde{s} + \tilde{n}$

- For individual sources $\langle \tilde{s} \rangle \neq 0$
- For GWBs $\langle \tilde{s} \rangle = 0$
- For noise $\langle \tilde{n} \rangle = 0$

For an **isotropic GWB** $\longrightarrow \langle h_\lambda(\vec{k}) h_{\lambda'}^*(\vec{k}') \rangle \propto \delta_{\lambda\lambda'} P_h^\lambda(k) \delta(\vec{k} - \vec{k}')$

Assuming $\langle \tilde{s}\tilde{n} \rangle = 0$ and Gaussian signal and noise

$$\langle \tilde{d}^2 \rangle = \langle \tilde{s}^2 \rangle + \langle \tilde{n}^2 \rangle = \sum_{\lambda} \mathcal{R}_{\lambda} P_h^{\lambda} + N \equiv \mathcal{R} [P_h + S_n]$$

where we have introduced

- The **(quadratic) response function** of the instrument \mathcal{R}
- The (intensity of the) **signal power spectrum** P_h (in 1/Hz)
- The **noise power spectrum** N (in 1/Hz)
- The **(square of the) Strain sensitivity** S_n (in 1/Hz)

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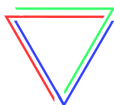
In order to compare with cosmological predictions it's customary to introduce

$$\Omega_{\text{GW}} \equiv \frac{1}{3H_0^2 M_p^2} \frac{\partial \rho_{\text{GW}}}{\partial \ln f} = \frac{4\pi^2}{3H_0^2} f^3 P_h \quad \text{and} \quad \Omega_n(f) = \frac{4\pi^2}{3H_0^2} f^3 S_n(f),$$

where $H_0 \simeq h_0 \times 3.24 \times 10^{-18}$ Hz is the Hubble parameter today.

Time Delay Interferometry

The simplest option is to build three **(correlated)** Michelson-like data streams

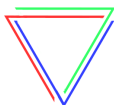


Laser frequency noise is **huge** and must be suppressed with **Time Delay Interferometry (TDI)**

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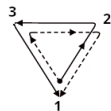
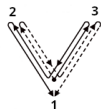
Using η_{ij} measurement in i coming from j and the delay operator D_{ij} we define:

- **Michelson variables**, dubbed **XYZ**, defined as (YZ are permutations):

$$X \equiv (1 - D_{13}D_{31})(\eta_{12} + D_{12}\eta_{21}) + (D_{12}D_{21} - 1)(\eta_{13} + D_{13}\eta_{31})$$

- **Sagnac variables**, dubbed **$\alpha\beta\gamma$** , defined as ($\beta\gamma$ permutations):

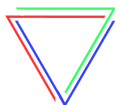
$$\alpha \equiv \eta_{12} + D_{12}\eta_{23} + D_{12}D_{23}\eta_{31} - (\eta_{13} + D_{13}\eta_{32} + D_{13}D_{32}\eta_{21})$$



Minimal working assumptions and their breakdown

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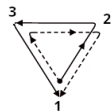
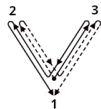
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In these variables, **signal and noise** (in different channels) **are correlated!**

For equal arms, diagonalization via:
(diagonal variables dubbed AET and \mathcal{AET})

$$\rightarrow \begin{pmatrix} -1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 1/\sqrt{6} & -2/\sqrt{6} & 1/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{pmatrix} \quad 7/31$$

Minimal working assumptions and their breakdown

The LISA noise model

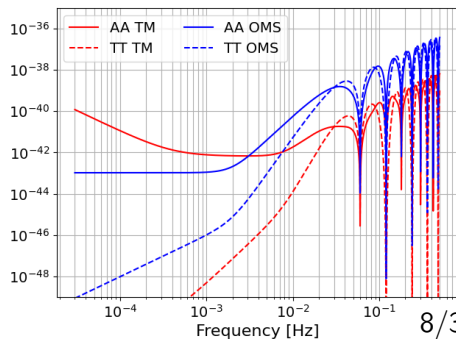
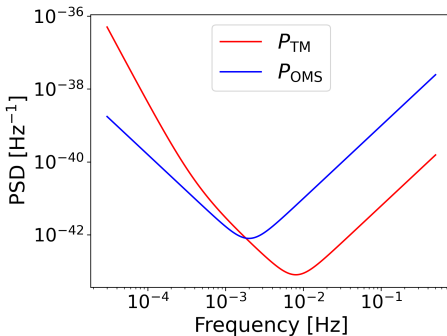
After TDI two main noise components:

Low frequencies are dominated by **Test Mass (TM)** noise
 large frequencies by **Optical Metrology System (OMS)** noise:

$$P_{TM}(f, A) = A^2 \times 10^{-30} \times F_{TM}(f),$$

$$P_{OMS}(f, P) = P^2 \times 10^{-24} \times F_{OMS}(f),$$

where $F_{TM}(f)$, $F_{OMS}(f)$ are some functions of frequency.



Minimal working assumptions and their breakdown

The LISA response function

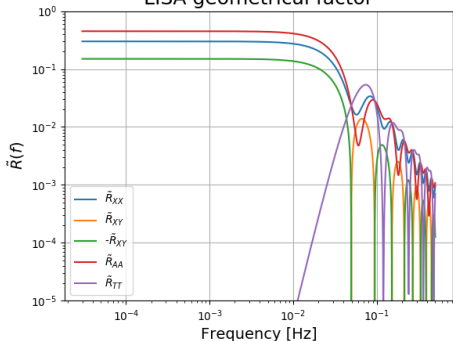
For an **isotropic** and **non-chiral spectrum** we get (see 2009.11845):

$$\langle s_i^{TDI} s_j^{TDI} \rangle = \int dk P_h(k) \mathcal{R}_{ij}^{TDI}(k), \quad \mathcal{R}_{ij}^{TDI}(k) \equiv 4 (2\pi kL)^2 |W^{TDI}(kL)|^2 \tilde{R}_{ij}^{TDI}(k).$$

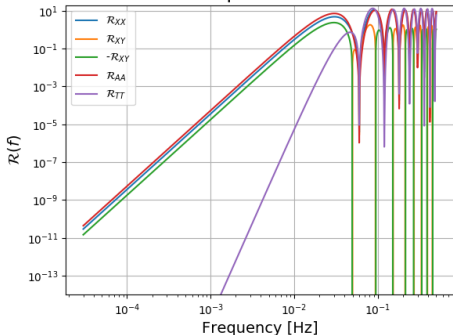
where $\mathcal{R}_{ij}^{TDI}(k)$ is the **LISA response** and $\tilde{R}_{ij}^{TDI}(k)$ is a geometrical factor.

For XYZ/AET (AET is \sim diagonal) combinations we get:

LISA geometrical factor



LISA response function



At low frequencies the TT response is suppressed by a factor f^6 !

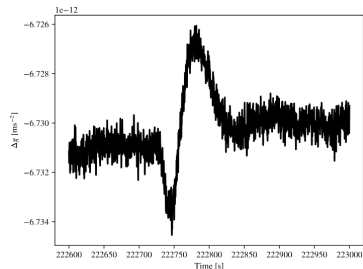
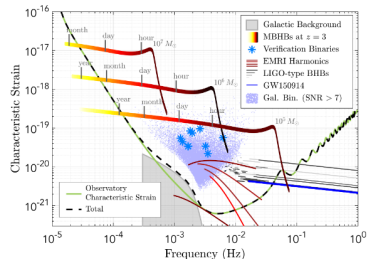
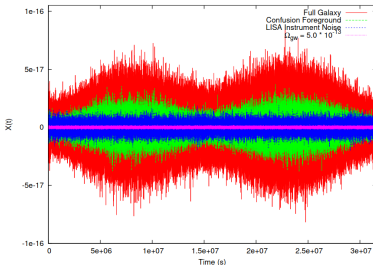
See https://github.com/Maupieronni/GW_response

Minimal working assumptions and their breakdown

Stationarity won't hold ...

The LISA data won't be stationary:

- There will be transient signals
- Measured GWBs change with time
- There will be transient glitches

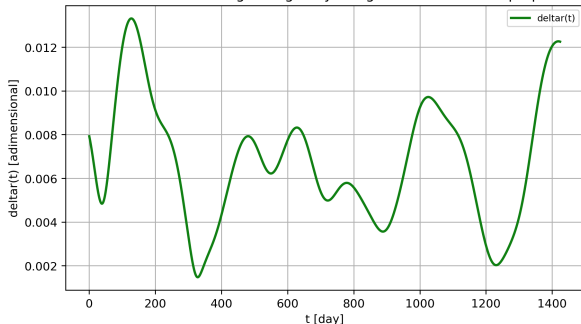


* LISA Collaboration, P. Amaro-Seoane et al., ArXiv: 1702.00786. M. R. Adams, Phys. Rev. D 89 (2014), 022001, ArXiv: 1307.4116. Q. Baghi et al., Phys. Rev. D 105 (2022), 042002, ArXiv: 2112.07490.

Minimal working assumptions and their breakdown

... LISA won't have equal arms...

time evolution of arm-length inequality using ESA orbits and SC proper time



Fluctuations in the arm-lengths of order up to 10^{-2} are expected!

Response functions and noise spectra will be modified



- Orthogonality of TDI variables might be affected
- T is not signal orthogonal anymore
- ...

An accurate description is necessary to avoid biases in the analysis!

... and noise characterization is not as simple!

Let us have a closer look at the problem of noise characterization
(still stick with TM and OMS noise only with known templates)

Each spacecraft contains two test masses and two lasers



12 (6 Acc + 6 OMS) independent noise components are expected!

$$\begin{pmatrix} A & 0 \\ 0 & P \end{pmatrix} \rightarrow \begin{pmatrix} A_{12} & 0 & 0 & D_{12}, A_{21} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & A_{23} & 0 & 0 & 0 & D_{23}, A_{32} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{31} & 0 & A_{21} & 0 & D_{31}, A_{13} & 0 & 0 & 0 & 0 & 0 \\ D_{12}, A_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & D_{23}, A_{23} & 0 & 0 & 0 & A_{32} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & D_{31}, A_{31} & 0 & 0 & 0 & A_{13} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{23} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{31} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{21} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{32} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{13} \end{pmatrix}$$

Several complications are added in the problem:

- Noise components propagate differently in different TDI variables
- Higher dimensionality of the parameter space
- Correlations between the noise parameters

Again, this requires care!

Simulation-Based-Inference (SBI)

Traditional methods (MCMC, nested sampling, whatever) are quite efficient and guaranteed to converge (in some cases)

but

scale poorly with number of parameters and require explicit likelihoods

Can alternative approaches perform better in some cases?

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Can alternative approaches perform better in some cases?

Normally, with Bayesian inference, we try to study the posterior probability:

$$p(\theta|d) = \frac{p(d|\theta) \pi(\theta)}{p(d)}$$

the idea of **SBI** is to use simulated (mock) data to build a **(ML-based) emulator for** (a part of) **the posterior!**

For example:

- **Neural Posterior Estimation (NPE)** directly targets $p(\theta|d)$
- **Neural Ratio Estimation (NRE)** targets $r(d, \theta)$ defined as

$$r(d, \theta) \equiv \frac{p(d|\theta)}{p(d)} = \frac{p(\theta|d)}{\pi(\theta)} = \frac{p(\theta, d)}{p(d) \pi(\theta)},$$

- ...

Basic ingredients and benefits of SBI

Key ingredients of the approach:

- A reasonable prior range for the parameters θ
- A simulator to generate mock data x_i given the parameters θ_i
- A neural network approximant, say $f_\phi(x, \bar{\theta})$, of the (unknow) target pdf
- A loss function \mathcal{L}_ϕ to minimize during training
- A training strategy/algorithm to optimize the parameters ϕ of the network

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Notice that:

- SBI does not require an explicit likelihood \rightarrow **likelihood-free inference**
- $f_\phi(x, \bar{\theta})$ is a smooth (infinitely differentiable function)
- $\bar{\theta}$ (target parameters), can be a subset of $\theta \rightarrow$ **automatic marginalization**
- SBI requires *independent* simulations \rightarrow fully profit of **GPU scalability**

There are good reasons to test its application ...

Neural Ratio Estimation (NRE)

As already mentioned:

$$r(d, \theta) \equiv \frac{p(d|\theta)}{p(d)} = \frac{p(\theta|d)}{\pi(\theta)} = \frac{p(\theta, d)}{p(d) \pi(\theta)},$$

i.e., $r(d, \theta)$ is the ratio between joint probability and marginal probability.

Given a pair (θ, d) , $r(d, \theta)$ can be used to assess whether θ can generate d !
The **network will be a classifier** to say whether θ, d are joint or marginal....

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The typical **loss function for NRE** is

$$\mathcal{L}[f_\phi] = - \int dx d\theta [p(x, \theta) \ln(\sigma(f_\phi(x, \theta))) + p(x)\pi(\theta) \ln(1 - \sigma(f_\phi(x, \theta)))],$$

where $\sigma(x) = (1 + e^{-x})^{-1}$ is the sigmoid function.

- **Integrals** are computed via **Monte Carlo integration**
- The **optimal classifier** $f_\phi^*(x, \theta)$ is precisely $f_\phi^*(x, \theta) = \ln r(x, \theta)$.

Let's see how this works in practice ...

A very simple example

Let's consider:

$$d = \theta^2 + \epsilon$$

with

$$\theta \sim U[-1, 1]$$

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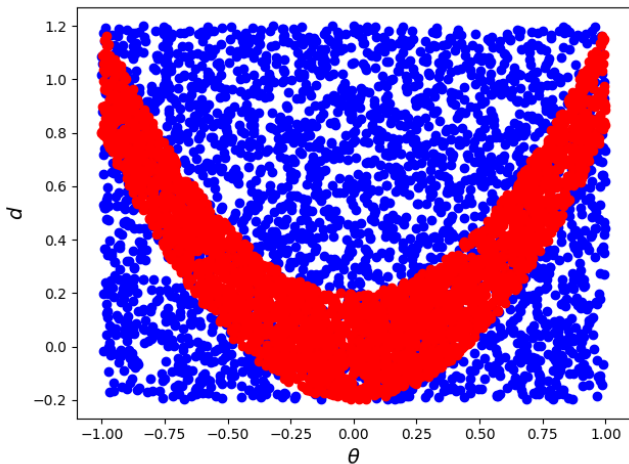
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Marginal samples

$$d, \theta \sim p(d)p(\theta)$$

Joint samples

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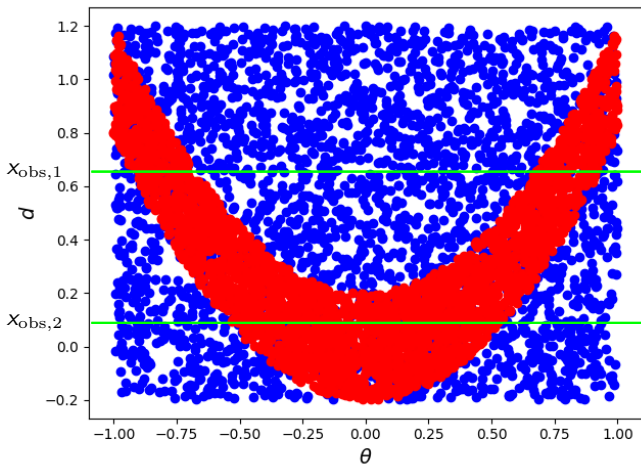
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Marginal samples

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In very large parameter spaces you can target a subset of the parameters!



Automatically marginalize over the other parameters (here ϵ)!

Neural Ratio Estimation (NPE)

The the **Kullback-Leibler (KL) divergence** is defined as

$$\text{KL}(P\|Q) \equiv \int P(\theta) \log \frac{P(\theta)}{Q(\theta)} d\theta.$$

measures the difference between two pdfs P and Q .

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The typical **loss function for NPE** is

$$\mathcal{L}(\phi) = - \int p(\theta, x) \log q_\phi(\theta|x) d\theta dx \simeq -\mathbb{E}_{p(\theta, x)}[\log q_\phi(\theta|x)],$$

- Again, the **integral** is computed via **Monte Carlo integration**
- Up to ϕ independent terms, $\mathcal{L}(\phi) = \text{KL}(p(\theta|x)\|q_\phi(\theta|x))$

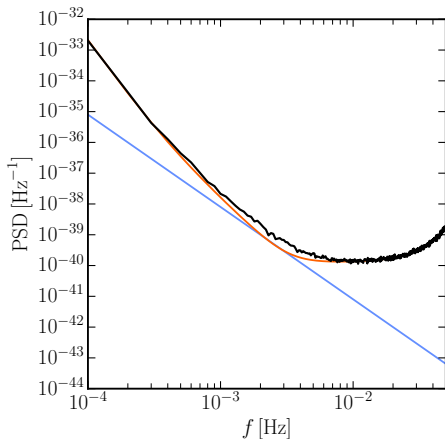
$$\begin{aligned} \mathcal{L}(\phi) &= \left(- \int p(\theta|x)p(x) \log q_\phi(\theta|x) d\theta dx \right) \\ &\simeq \int p(x) \left(\int p(\theta|x) \log \frac{p(\theta|x)}{q_\phi(\theta|x)} d\theta \right) dx \simeq \mathbb{E}_{p(x)} [\text{KL}(p(\theta|x)\|q_\phi(\theta|x))], \end{aligned}$$

where $p(x) = \int p(x|\theta)\pi(\theta)d\theta$ is the marginal data distribution.

Thus, $q_\phi(\theta|x)$ is an approximant for the full posterior $p(\theta|x)$

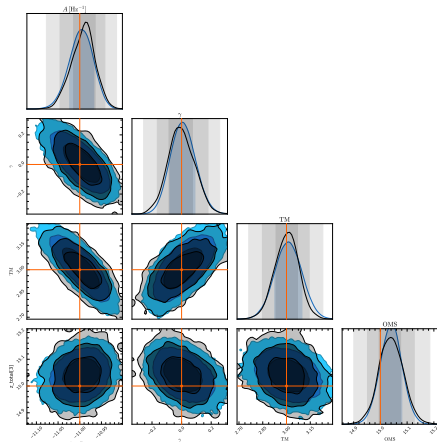
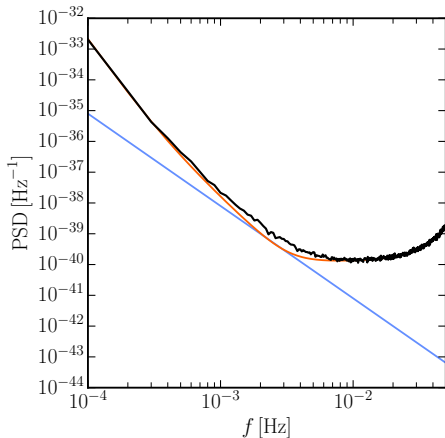
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Can we recover it with the same level of accuracy?



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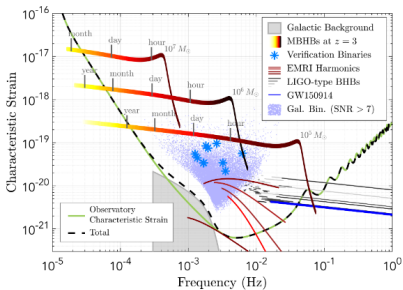
Good news!

James Alvey et al., Phys. Rev. D 109 (2024) 083008, ArXiv:2309.07954.

Code available at <https://github.com/PEREGRINE-GW/saqqara/>

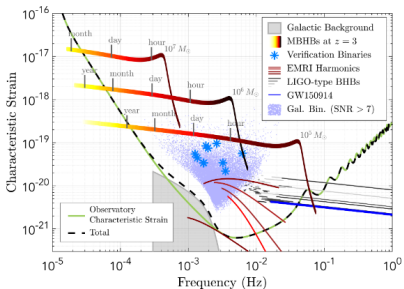
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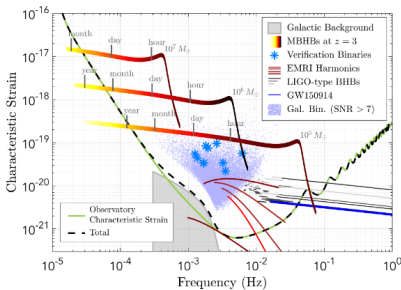


A **minimal setup** for this:
randomly inject transients slightly
below the threshold for detection

Would this still work??

... plus something completely new!

What if there's something else
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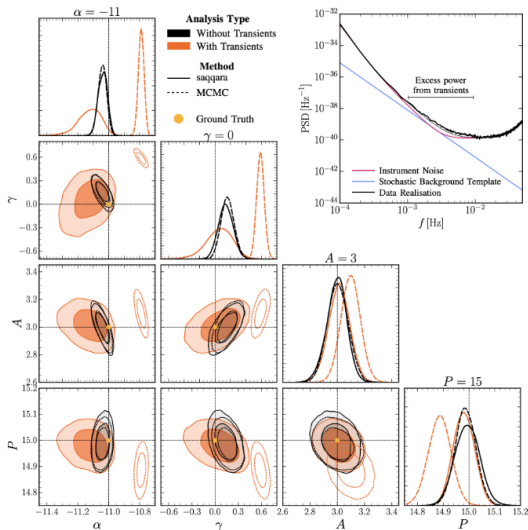
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Yes!!

James Alvey et al., Phys. Rev. D 109 (2024) 083008, ArXiv:2309.07954.

Code available at <https://github.com/PEREGRINE-GW/saqqara/>



What about noise non-stationarities? (FIM)

The **noise won't be stationary** for the whole mission duration ...

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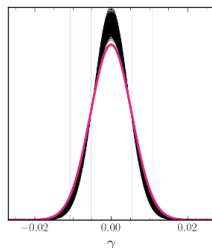
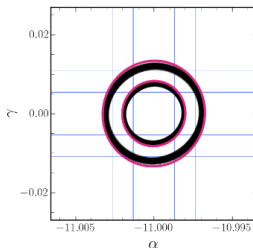
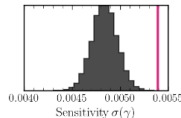
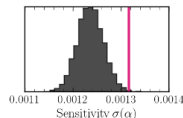
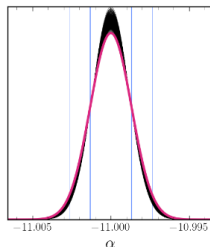
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First test with FIM:

Looks like you actually do better!

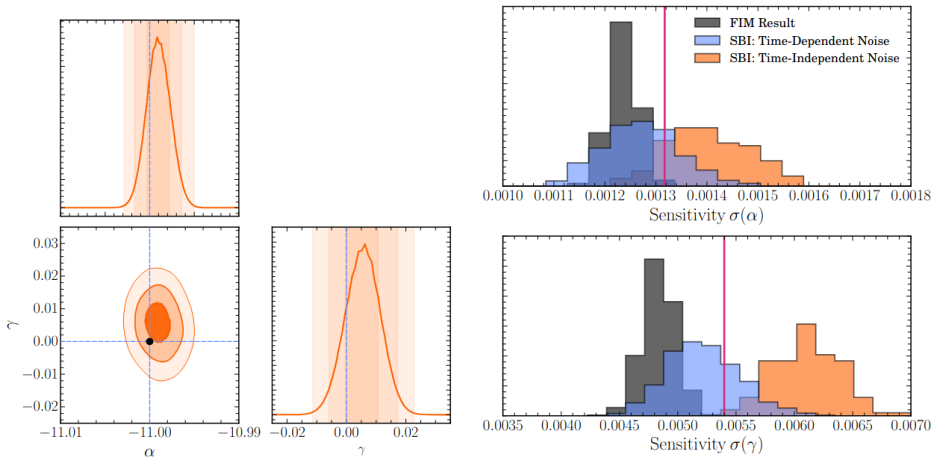


James Alvey et al., ArXiv:2408.00832.

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See also https://github.com/Mauropieronni/GW_response

What about noise non-stationarities? (Full analysis)

Validate FIM results with a full analysis



Moreover, training segment-by-segment is 100 times faster!

James Alvey et al., ArXiv:2408.00832.

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A convergence criterion for SBI

Ensamble learning for SBI

Can we assess the convergence / test model mis-specifications?

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In ML context this is known as **ensamble learning**

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Some observations:

- Checking the consistency between the networks is great for diagnostics
- KL is a natural choice for this ... and brings different info from the loss
- KL can be mapped into (interpreted as) σ levels
- Train until all KLs are below some value ensures consistency
- With an ensamble we can test model mis-specifications!

KL to assess convergence

Let's consider 1d Gaussians

expressing $\sigma_2 = \sigma_1(1 + \epsilon)$ and $\mu_2 = \mu_1 + \gamma\sigma_1$ with $\epsilon, \gamma \ll 1$ we get

$$\text{KL}(\mathcal{N}_1 \parallel \mathcal{N}_2) \simeq \frac{\gamma^2}{2} + \epsilon^2 - \gamma^2\epsilon$$

pretty clear scalings with ϵ and γ + can be generalized to n dimensions!

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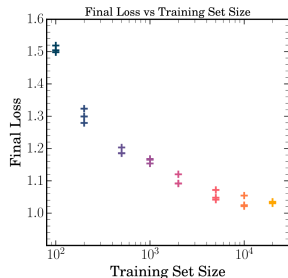
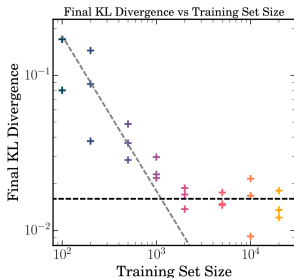
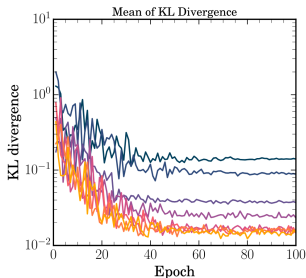
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— 100 — 200 — 500 — 1000 — 2000 — 5000 — 10000 — 20000 --- $1/n_{\text{Train}}$

KL brings **extra information compared to the loss function!**

In collaboration with James Alvey and Carlo Contaldi, ArXiv:2507.134955.

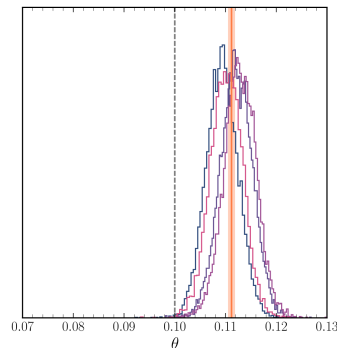
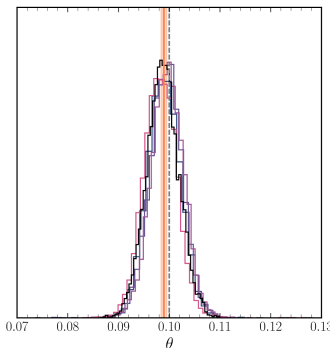
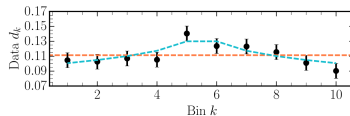
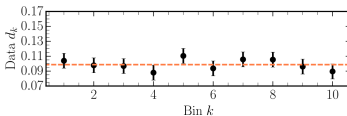
Test model mis-specifications I

Consider $d_k \sim \mathcal{N}(\mu_k, \sigma_k)$, with $\sigma_k = 0.01$ and $\mu_k = \theta + a\beta_k$
Train for $a = 0$, then apply on data with $a \neq 0$, what happens?

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Posterior (Network 1)
 Posterior (Network 2)
 Posterior (Network 3)
 Posterior (Network 4)
 Truth

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A convergence criterion for SBI

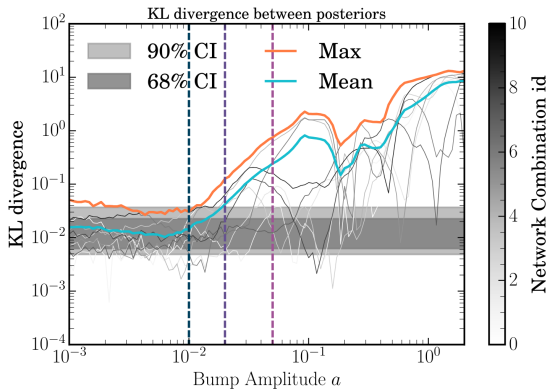
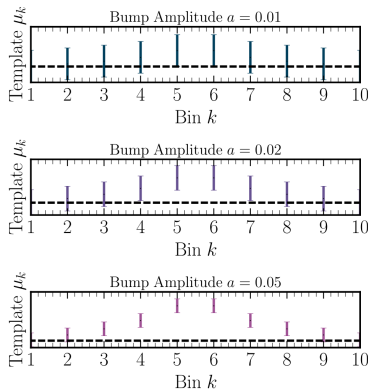
Test model mis-specifications II

How does KL increase with a (i.e., increasing the bump height)?

A convergence criterion for SBI

Test model mis-specifications II

How does KL increase with a (i.e., increasing the bump height)?



KL differences start to be grow for $a \sim 1\sigma$ deviations!

Dynamical (Round-free) Sequential SBI

Dynamical (Round-free) Sequential SBI: the general idea

Problem: Amortization struggles for large posterior to prior ratios.

To overcome this issue, we can use **Sequential SBI**

Key idea: use the posterior from the round i to update the prior for round $i + 1$.

Main drawback: requires some criterion to decide when to stop one round.

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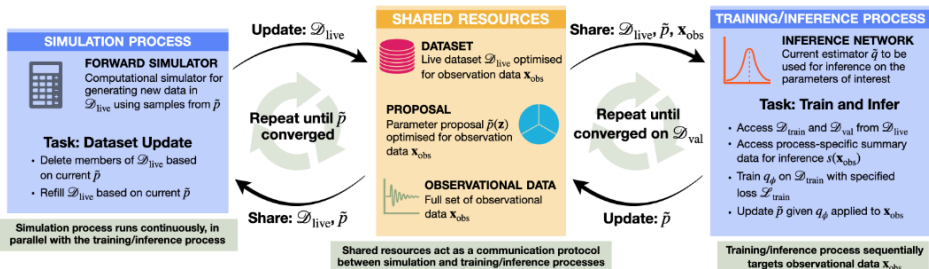
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DYNAMIC SIMULATION-BASED INFERENCE



H. Lyu, et al., ArXiv: 2510.13997.
Code available at <https://github.com/dynamic-sbi/chameleon-sbi> and
github.com/dynamic-sbi/falcon-dsbi/

Dynamical (Round-free) Sequential SBI: an example run

Does it work for some
non-trivial problems?

Consider a 10d problem with
complex posterior structure
i.e., multi-modality

+

large posterior to prior ratio
 $10^4/10^{-2}$ per dimension,
i.e. $\sim 10^{60}$

Can we recover the posterior?

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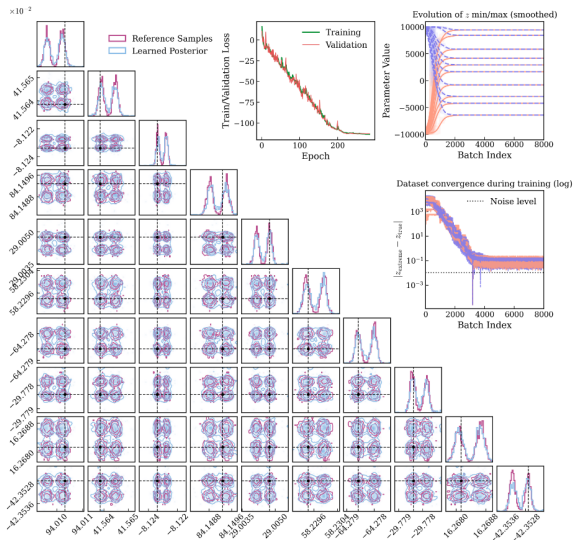
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Looks so!

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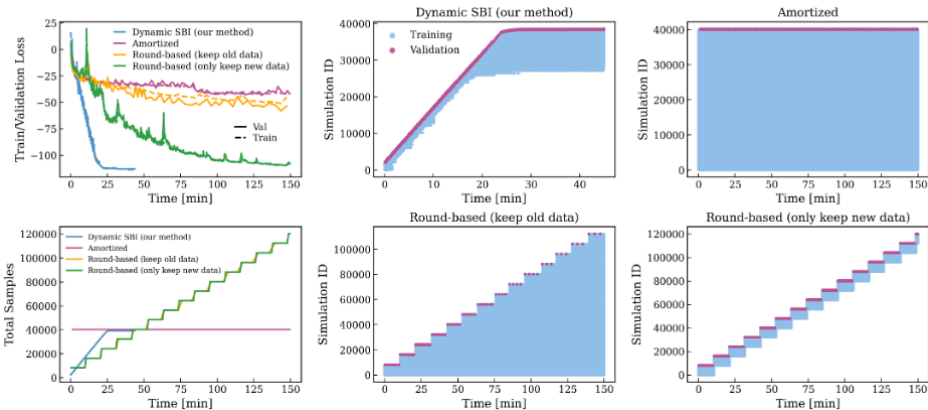


Dynamical (Round-free) Sequential SBI

Dynamical (Round-free) Sequential SBI: benchmark test

... great but does it work better?

Let's compare it with other training strategies:



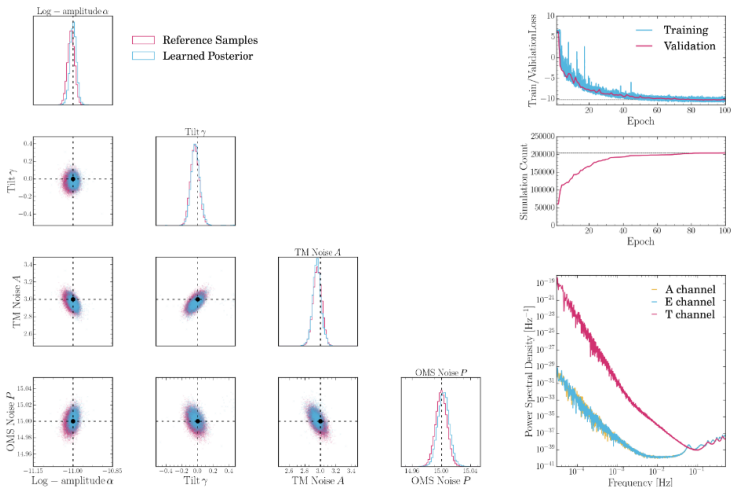
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Dynamical (Round-free) Sequential SBI

Dynamical (Round-free) Sequential SBI: SGWB with LISA

Does it work well for “real life” problems? e.g., SGWB with LISA data?



H. Lyu, et al., ArXiv: 2510.13997. Code available at <https://github.com/dynamic-sBI/chameleon-sbi> and github.com/dynamic-sBI/falcon-dsbi/

Conclusions and outlook

Conclusions

- Analyzing LISA data will be quite difficult
- Traditional methods work (but they might be expensive)
- SBI (and ML in general) looks quite promising ...
- ... it works with transients and non-stationarities in the noise!
- Some convergence criteria can be quite useful

Future perspectives

- Keep improving on detector modeling
- More realistic noise model
- Faster data generation procedure?
- More complexity in the simulator
- New ideas/techniques?
- ...

Last slide

The end

Thank you for your attention!