Classical time evolution in presence of ghosts: From scalar models to black-hole binaries

Aaron Held

Philippe Meyer Junior Research Chair
École Normale Supérieure

June 19, 2025: AstroParticle Physics Seminar
SISSA, Trieste, Italy

Motivation: Gravitational Effective Field Theories

Part I: Ghostly interactions in classical field theory

with Cédric Deffayet, Shinji Mukohyama, and Alexander Vikman (2023, 2025)

Part II: Well-posed time evolution & black hole binaries

with Hyun Lim (2021, 2023, 2025)
with Pau Figueras and Aaron Kovacs (2024), also ongoing with Ramiro Cayuso

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The Effective Field Theory Framework ...

Assuming a given

(i)	IR **field content**	**metric / curvature ***
(ii)	IR **symmetries**	**Lorentz invariance**
(iii)	**expansion** scale/scheme	**derivative / curvature**

we expand the effective action in all possible operators.

*hidden additional gravitational degrees of freedom

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... systematically captures modifications of GR.

Joint derivative/curvature expansion ...

$$\mathcal{L}^{(1)}_{\mathsf{EFT}} = \mathsf{M}^2_{\mathsf{Pl}}\,\mathsf{R}$$

$$\mathcal{L}^{(2)}_{\mathsf{EFT}} = \left[\alpha_0 \mathsf{R}_{\mathsf{ab}}\mathsf{R}^{\mathsf{ab}} - \beta_0\,\mathsf{R}^2\right]$$

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After reduction via

- (i) index symmetries
- (ii) geometric identities
- (iii) 4D-specific identities (e.g. Gauss-Bonnet)

see Fulling CQG 9 (1992); Martin-Garcia, Yllanes, Portugal, CPC 179 (2008)

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order-by-order field redefinitions of the form

$$g_{ab} \to g_{ab} + c_1\, g_{ab}\, X + c_2\, X_{ab}$$

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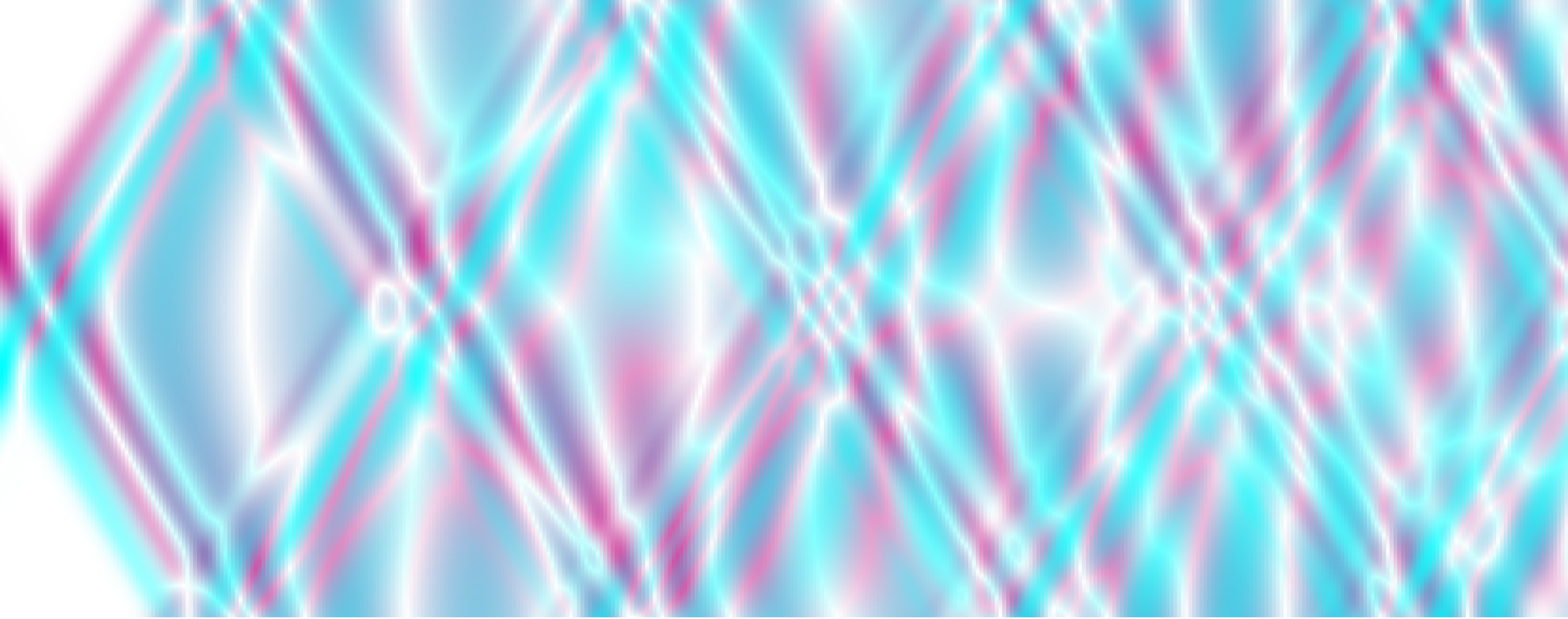

Part I: Ghostly interactions in classical field theory

Deffayet, Held, Mukohyama, Vikman, JCAP 11 (2023) 031
Deffayet, Held, Mukohyama, Vikman, 2504.11437

The **Hamiltonian** of all higher-derivative non-degenerate classical point-particle theories is **unbounded from above and below.**

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Point-particle models can be stable if the potential at large phase-space distance is **dominated by stable self-interactions.**

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Point-particle systems w/ opposite-sign kinetic terms ...

Integrable Liouville models ...

$$H_{LV} = \frac{p_x^2}{2} + \sigma \frac{p_y^2}{2} + V_{LV}(x, y)$$

$$V_{LV} = \frac{f(u) - g(v)}{u^2 - v^2}$$

$$u^2 = 1/2 \left(r^2 + c + \sqrt{(r^2 + c)^2 - 4\, c\, x^2} \right)$$

$$v^2 = 1/2 \left(r^2 + c - \sqrt{(r^2 + c)^2 - 4\, c\, x^2} \right)$$

$$r^2 = x^2 + \sigma\, y^2$$

Deffayet, Mukohyama, Vikman, PRL 128 (2022) 4
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... can nevertheless be globally (Lagrange) stable.

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... are Lagrange stable if ...

(i) ... f(u) and g(v) are bounded below, i.e.,

$$f(u) \geqslant f_0 \quad \& \quad g(v) \geqslant g_0$$

(ii) ... at large |u| and |v|, these bounds sharpen to

$$f(u) \geqslant 4F_0\, |u|^\zeta > 0 \quad \& \quad g(v) \geqslant 4G_0\, |v|^\eta > 0$$

with $f_0, g_0 \in \mathbb{R}\,, \quad F_0, G_0 \in \mathbb{R}^+\,, \quad \zeta > 2\,,\ \eta > 2$

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... contain a polynomial subclass ...

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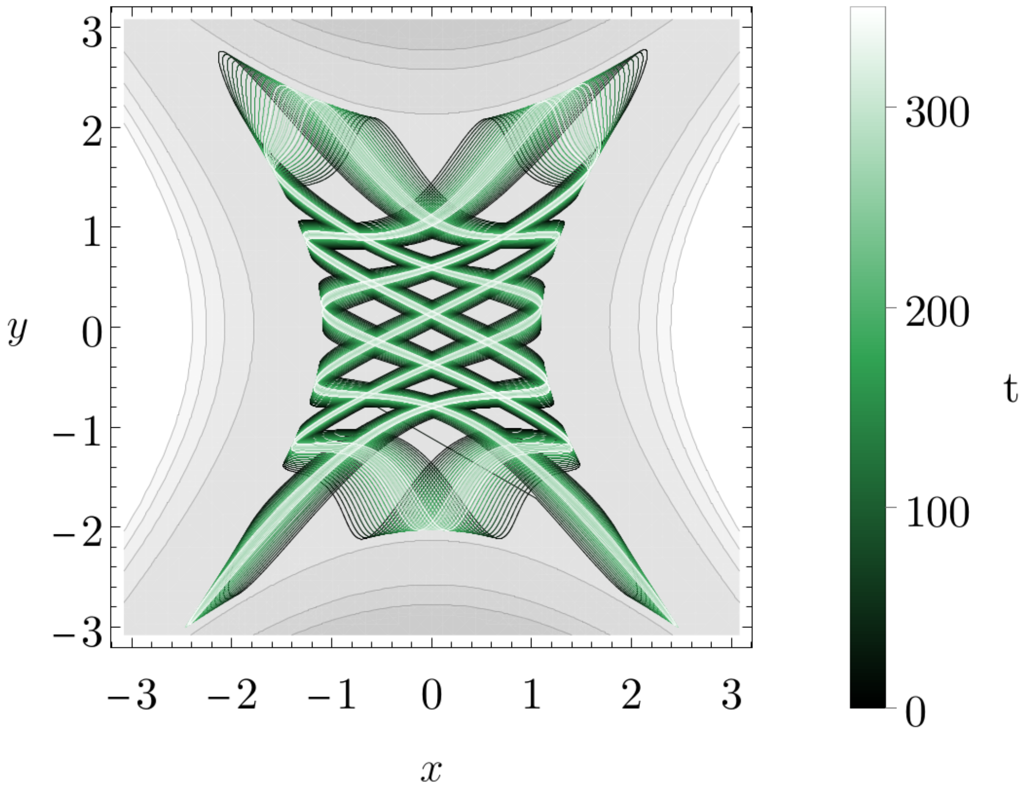

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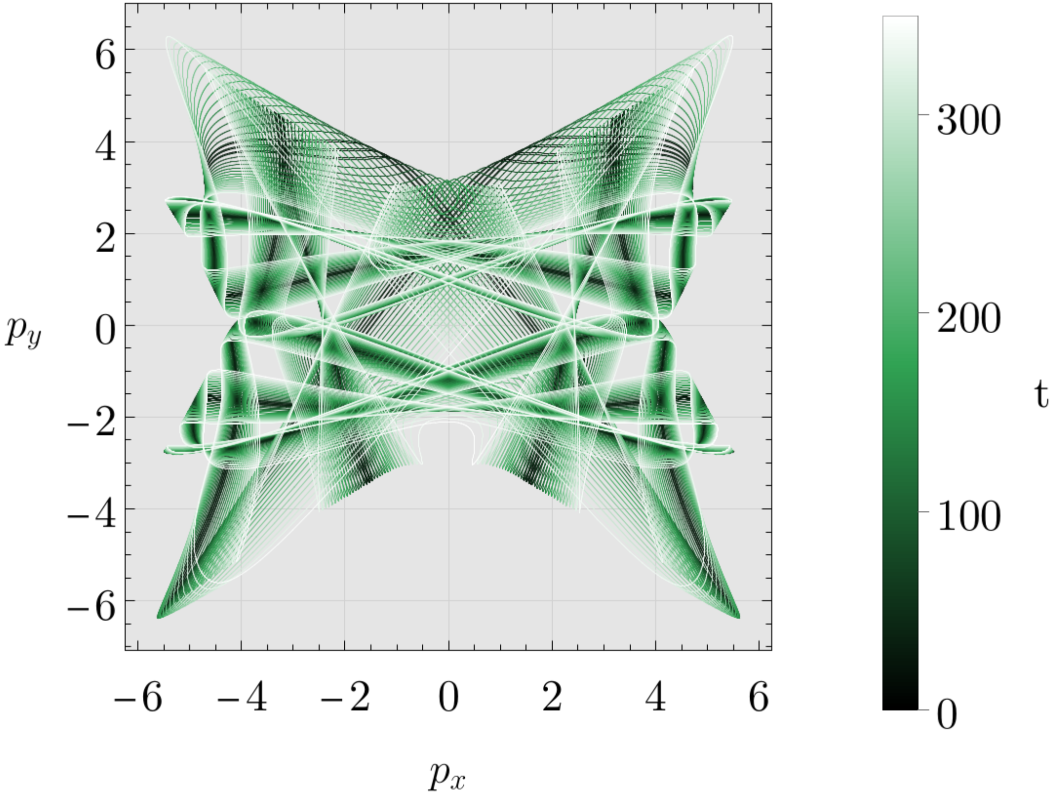

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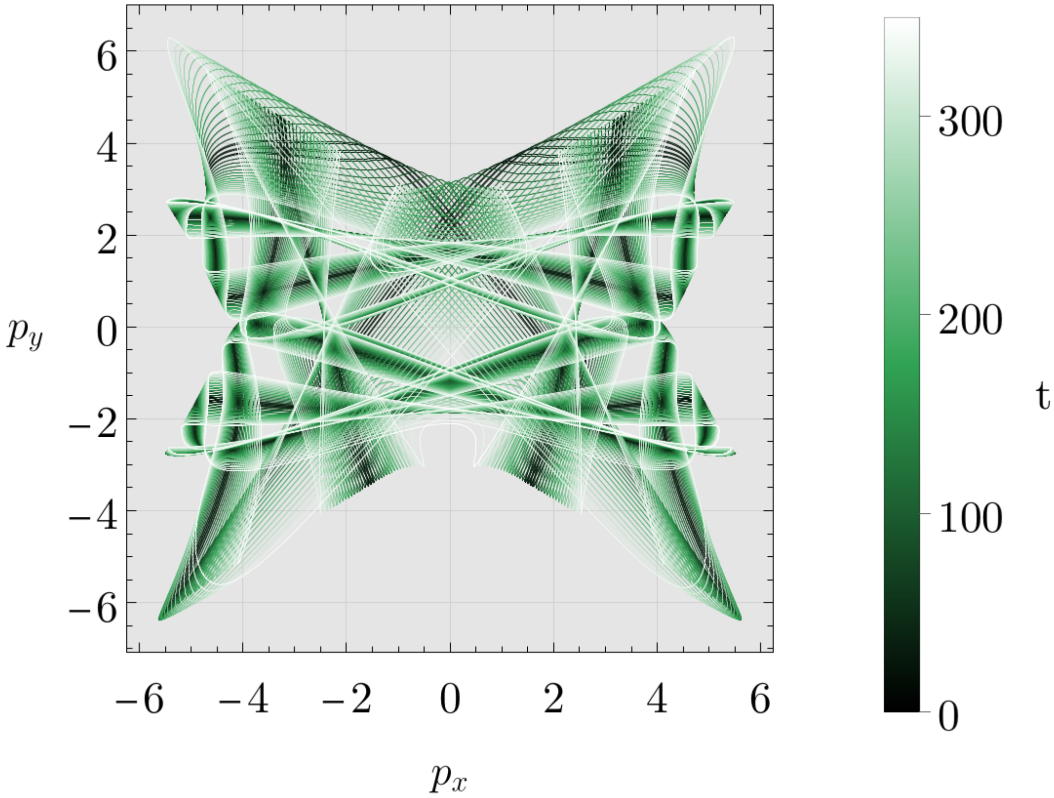

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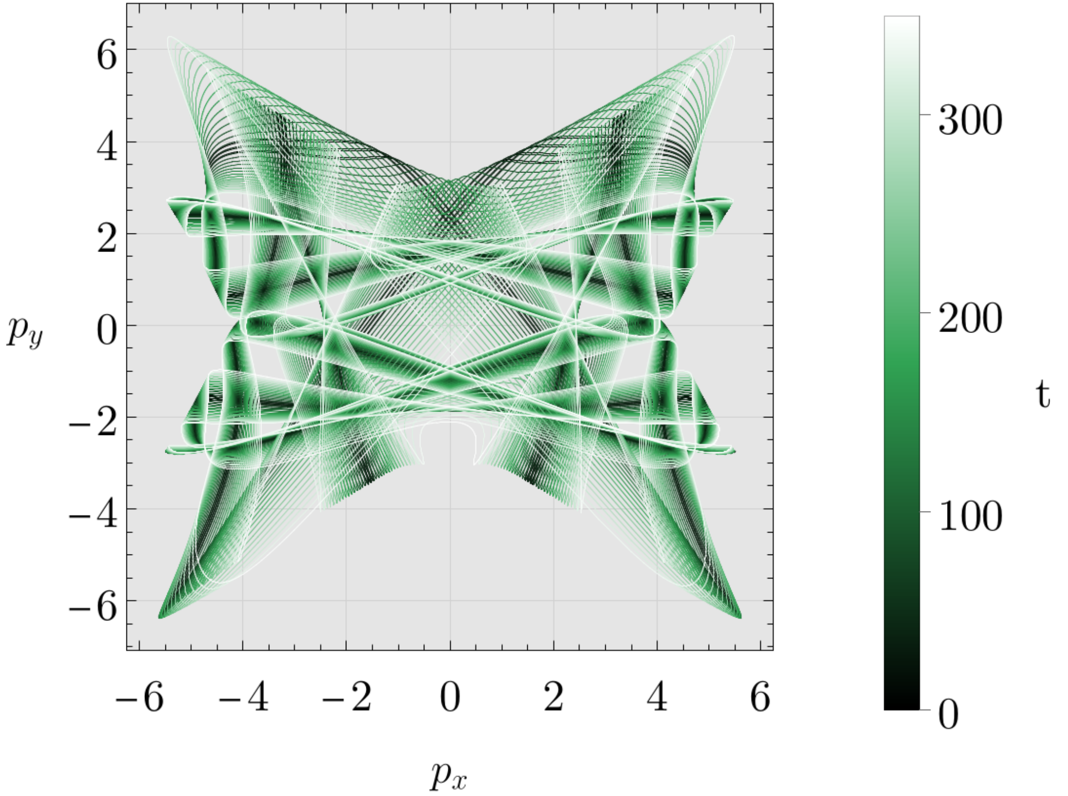

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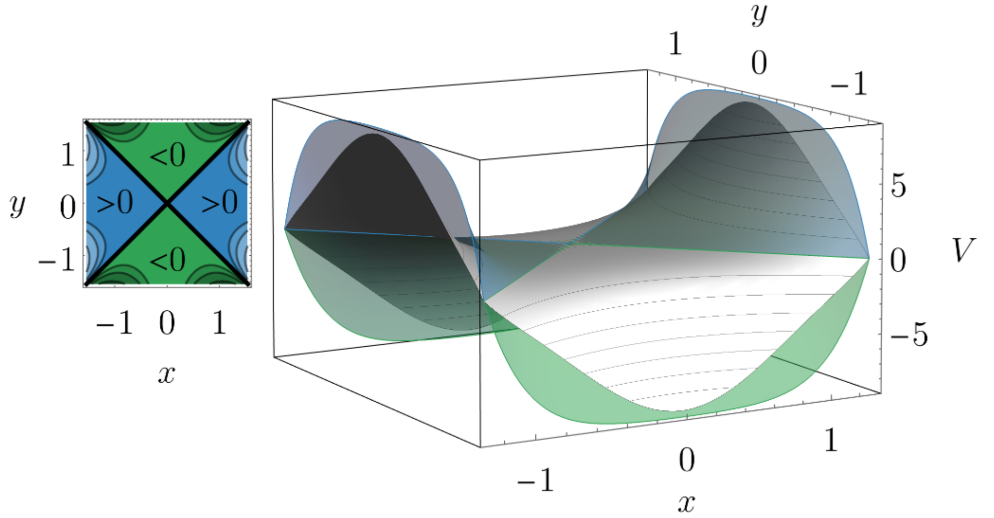
Point-particle systems w/ opposite-sign kinetic terms …

… identify **conditions on the potential**

- decoupled potentials are individually stable
- At large |x|, |y|, the full potential is dominated by the decoupled potentials
  - $V(x,y) \lesseqgtr V_P(x) + V_G(y) \quad \forall\ |x| \gtrless |y|$
  - For |x|=|y|, the Hessian converges to a stable decoupled Hessian



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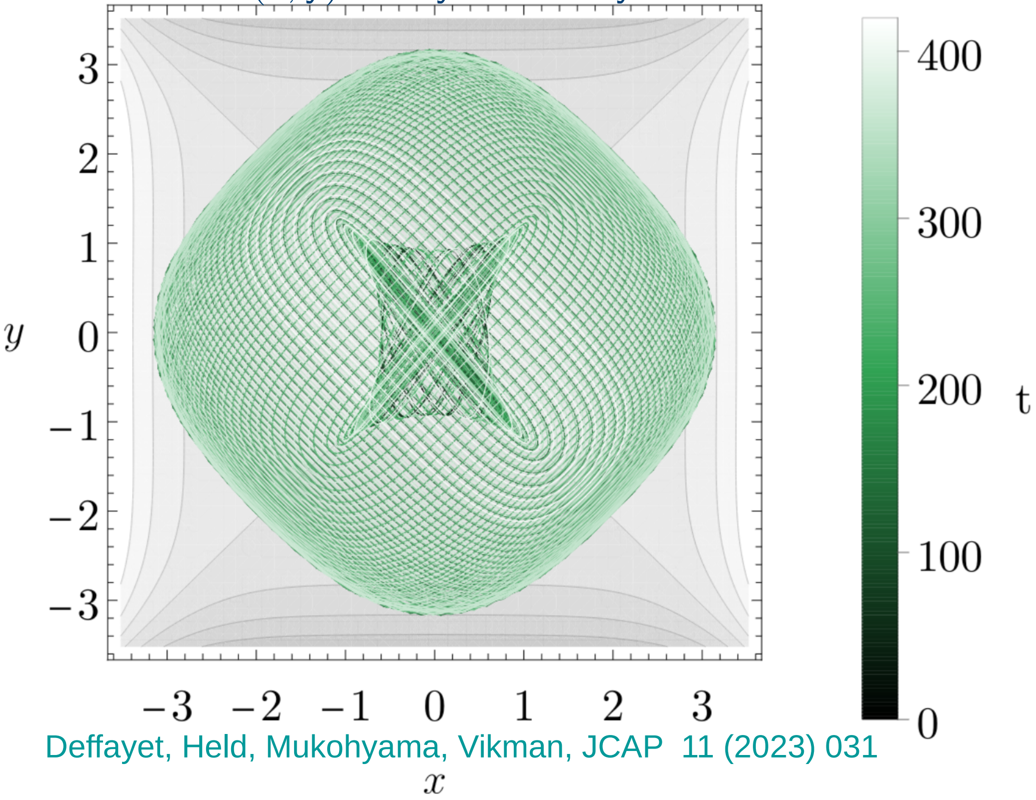

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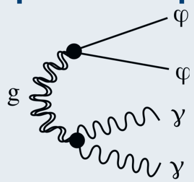
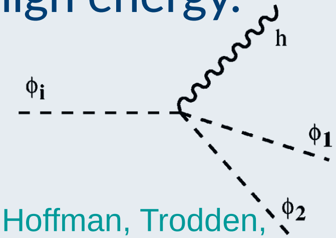


Cline, Jeon, Moore, PRD 70 (2004)



Caroll, Hoffman, Trodden, PRD 68 (2003)

Part I: Ghostly interactions in classical field theory

Deffayet, Held, Mukohyama, Vikman, JCAP 11 (2023) 031
Deffayet, Held, Mukohyama, Vikman, 2504.11437

The **Hamiltonian** of all higher-derivative non-degenerate classical point-particle theories is **unbounded from above and below.**

Ostrogradski 1857

Point-particle models can be stable if the potential at large phase-space distance is **dominated by stable self-interactions.**

Deffayet, Held, Mukohyama, Vikman, JCAP 11 (2023) 031

Classical field theories do not decay instantaneously and can **exhibit longlived motion.**

Deffayet, Held, Mukohyama, Vikman, 2504.11437

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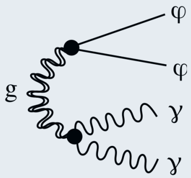

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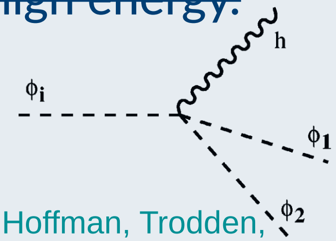

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Urries, Julve J. Phys. A 31 (1998)

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$$\mathcal{L} = -\frac{1}{2}\phi\left[\Box + \mathrm{m}_\phi^2\right]\phi - \frac{\sigma}{2}\chi\left[\Box + \mathrm{m}_\chi^2\right]\chi - \mathrm{V}(\phi,\,\chi)$$

σ = + 1: non-ghostly
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... do not obstruct from well-posedness.

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(non-)ghostly nature (σ = ± 1)
does not affect the principal part

... cannot lead to instantaneous classical decay.

Numerical solution …

Numerical solution …

julia

- using DifferentialEquations.jl
  https://github.com/aaron-hd/ghostlyPDE_1D
- julia-based PDE solver
  Rackauckas, Nie, JORS 5 (2017)

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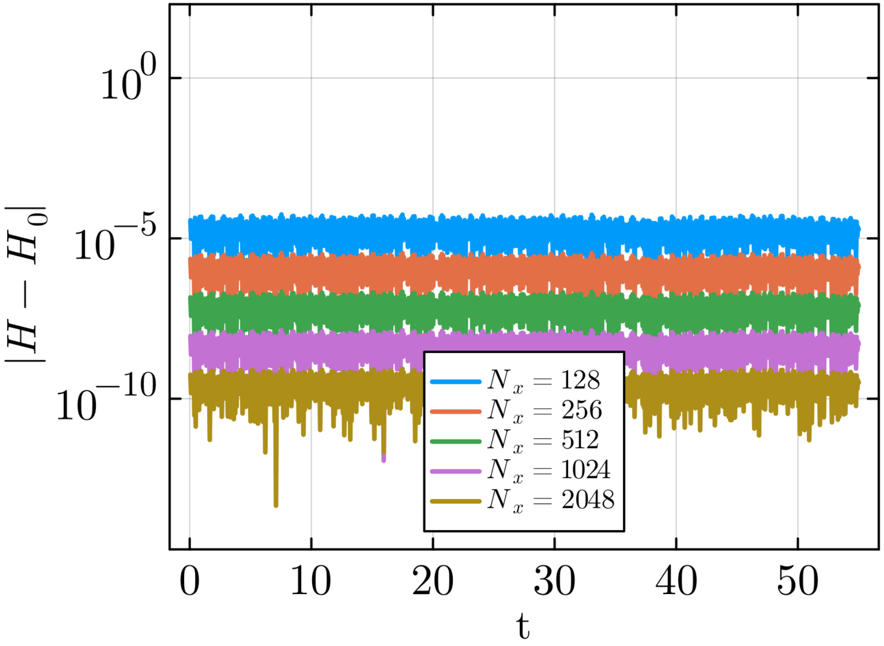

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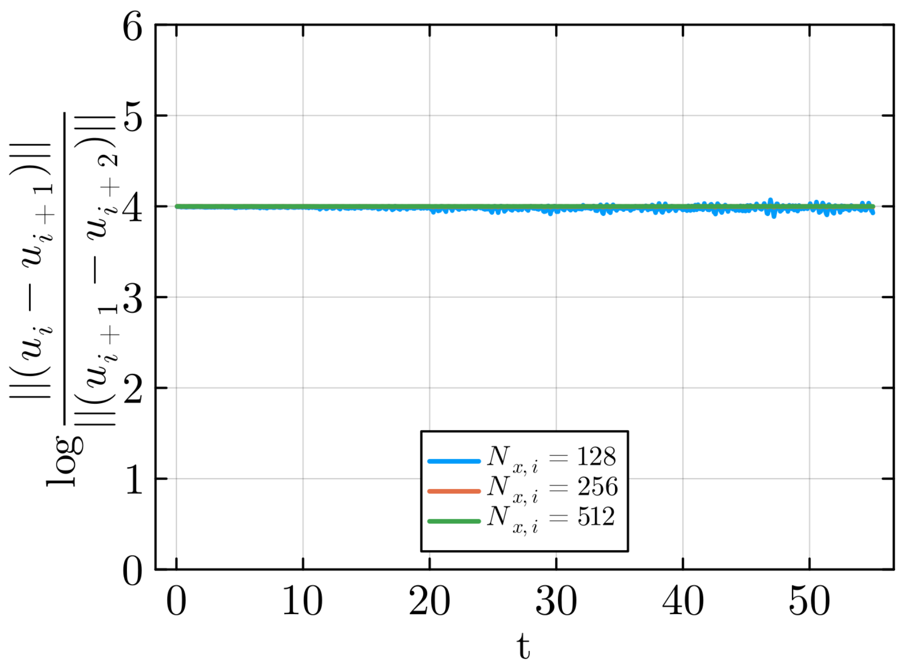

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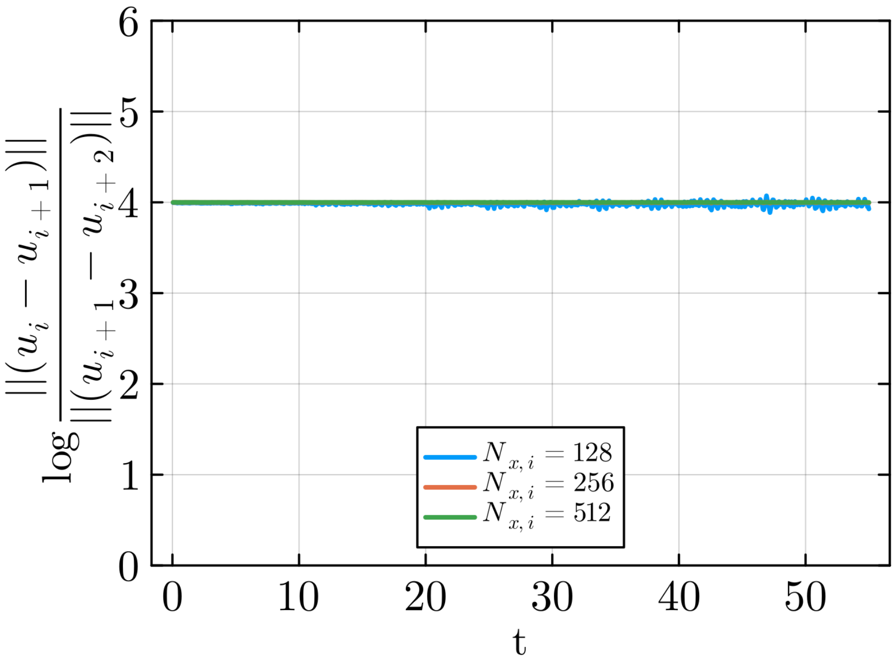

... converges to the continuum field theory.

Unquenched instability ...

Deffayet, Held, Mukohyama, Vikman, 2504.11437

$$V = \lambda_{nm}\,\phi^n \chi^m$$

(1+1)D Simulation converges to the solution of the continuum field theory

- 4^{th} order FD
- 4^{th} order RK4 timestep
- self-convergence rate verified at all times

Model parameters

- $n = m = 2$
- $\lambda L^2 = 1$;
- $m_\phi L = m_\chi L = 0$;

plane-wave initial data

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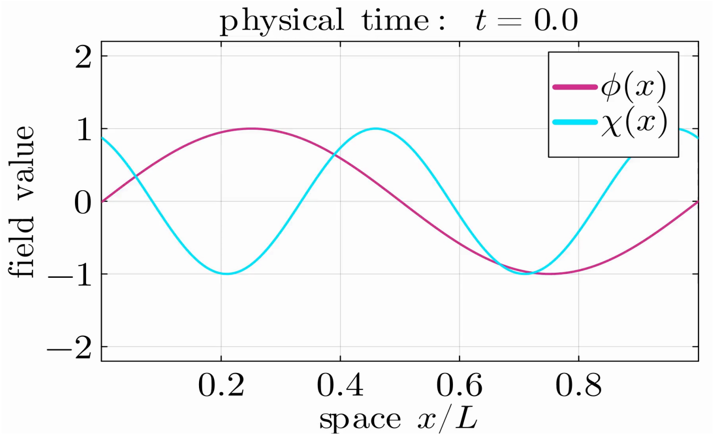

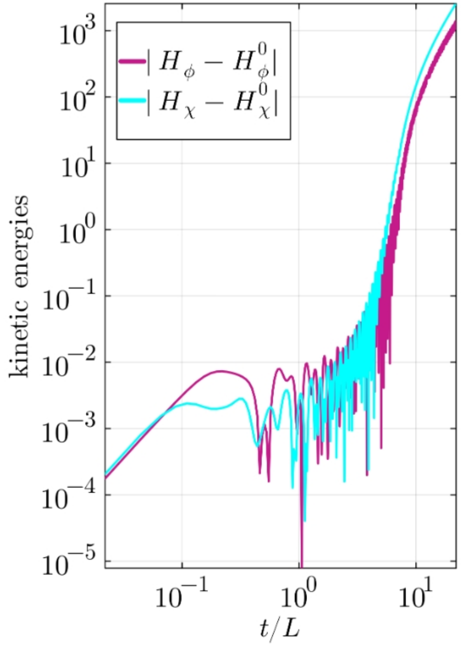

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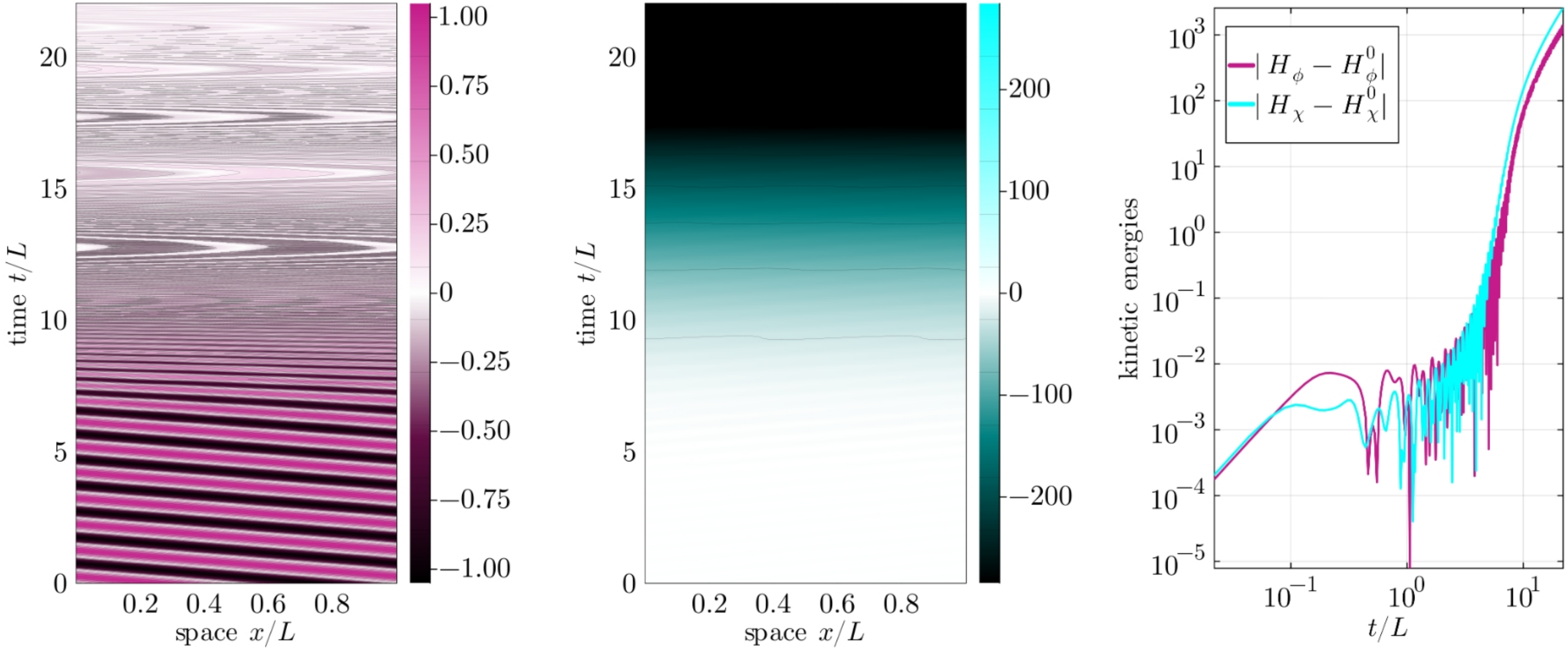

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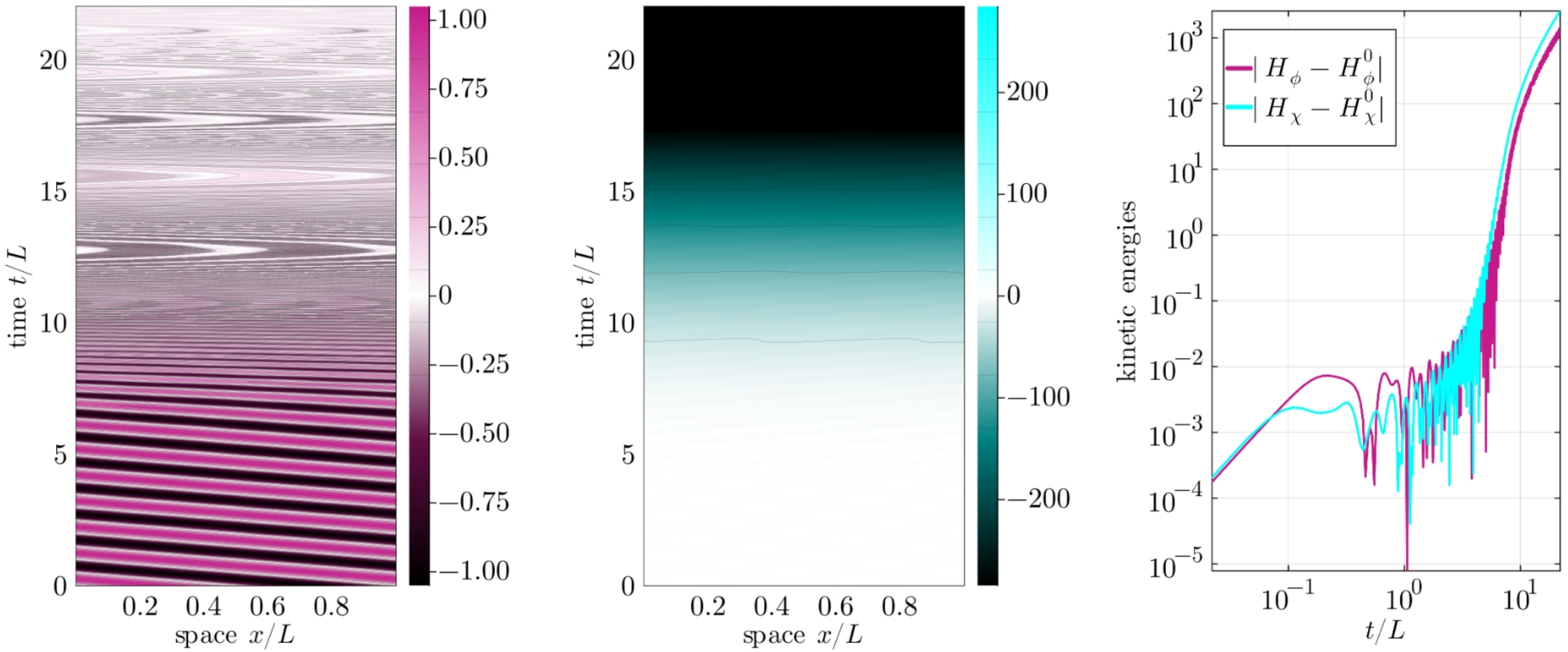

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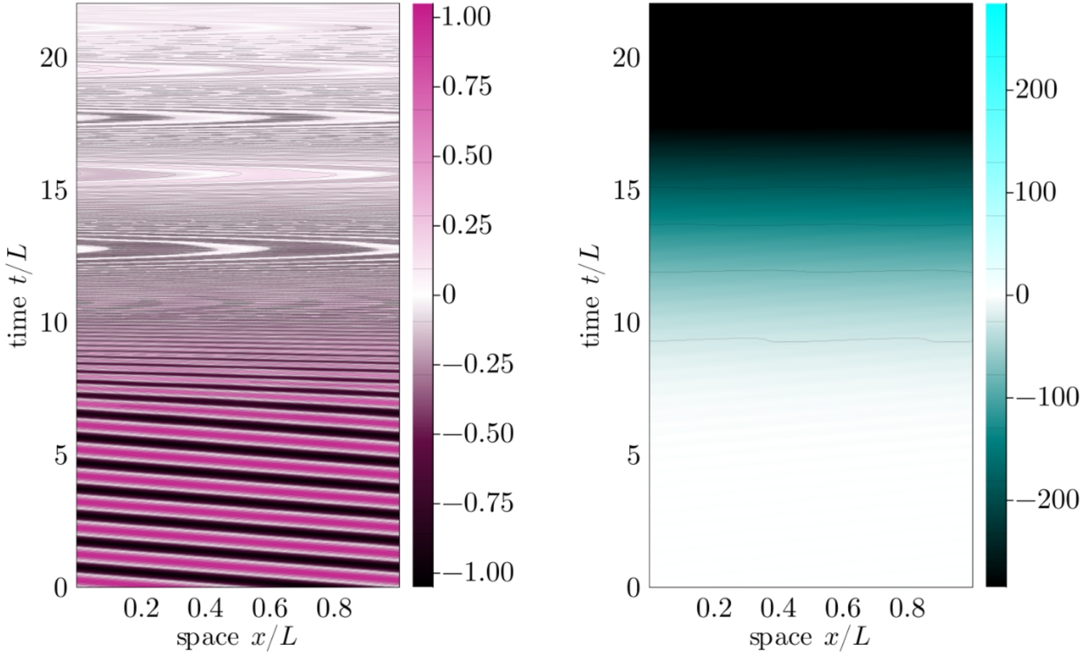

$$\Box\phi = -\left(m_\phi^2 + 2\,\lambda_{22}\,\chi^2\right)\phi \equiv -m_{\phi,\mathrm{eff}}^2\,\phi$$

$$\Box\chi = -\left(m_\chi^2 + 2\,\sigma\,\lambda_{22}\,\phi^2\right)\chi \equiv -m_{\chi,\mathrm{eff}}^2\,\chi$$

- no ghost: both effective masses positive
- with ghost: one effective mass positive
one effective mass negative

... can be benign.

Higher frequencies ...

Deffayet, Held, Mukohyama, Vikman, 2504.11437

$$V = \lambda_{nm}\,\phi^n \chi^m$$

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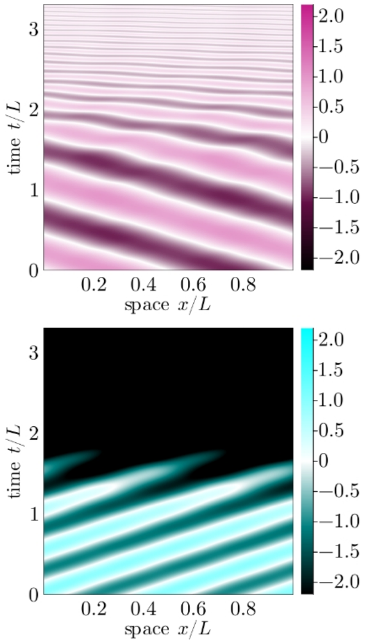

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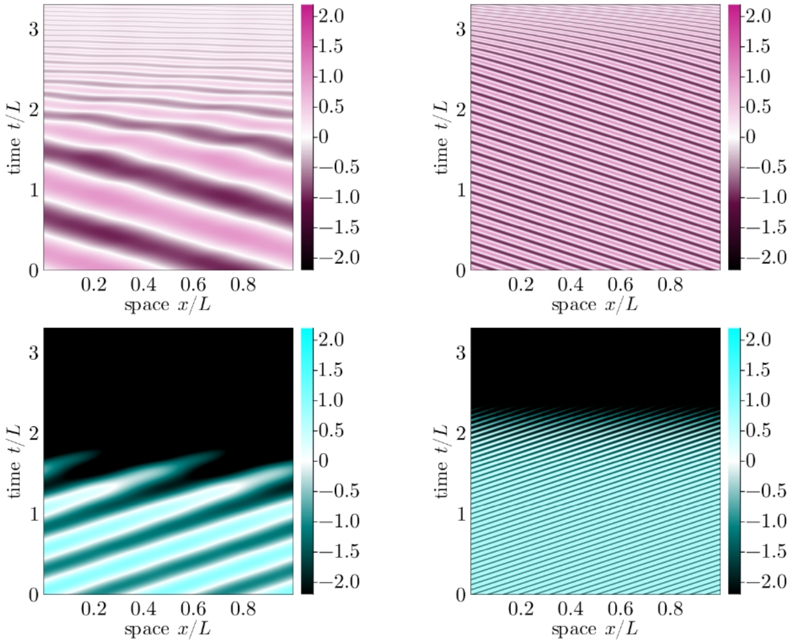

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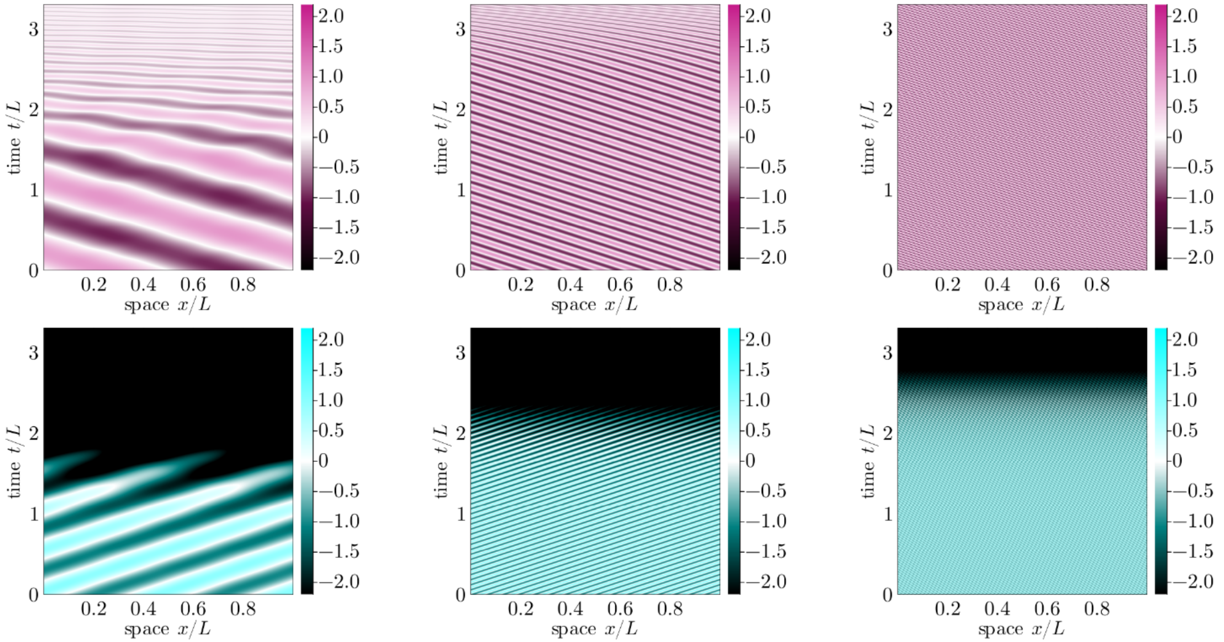

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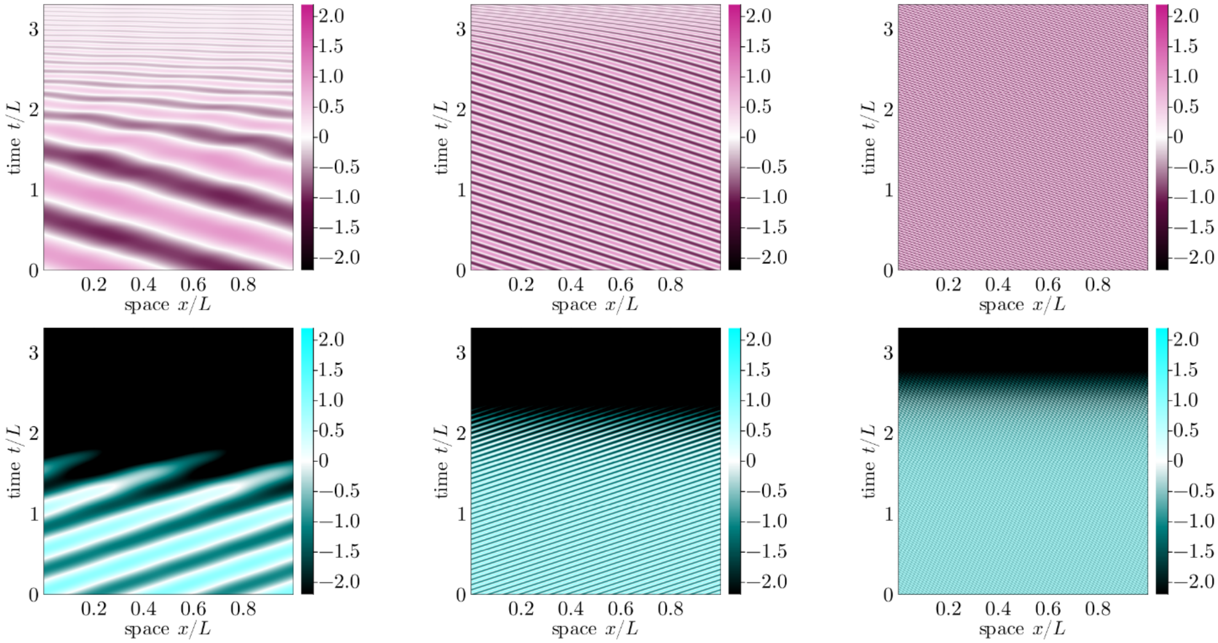

Deffayet, Held, Mukohyama, Vikman, 2504.11437

... are more stable, not less stable.

Opposite-sign kinetic terms …

Deffayet, Held, Mukohyama, Vikman, 2504.11437

Increasingly longlived for:

- lower initial amplitude
- higher initial frequency

initial data parameters

- weaker ghostly coupling
- larger masses

model parameters

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Deffayet, Held, Mukohyama, Vikman, JCAP 11 (2023) 031

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Model parameters setup in trivial vacuum

$$m_\phi^2 \equiv m_\chi^2 \equiv m^2$$

Various initial data families

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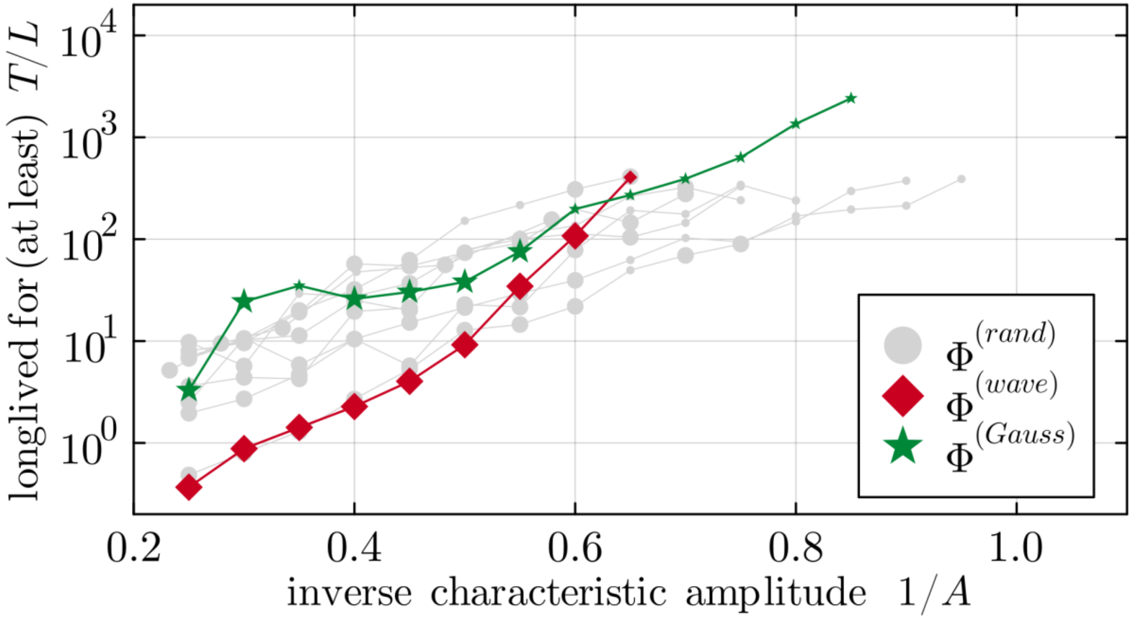

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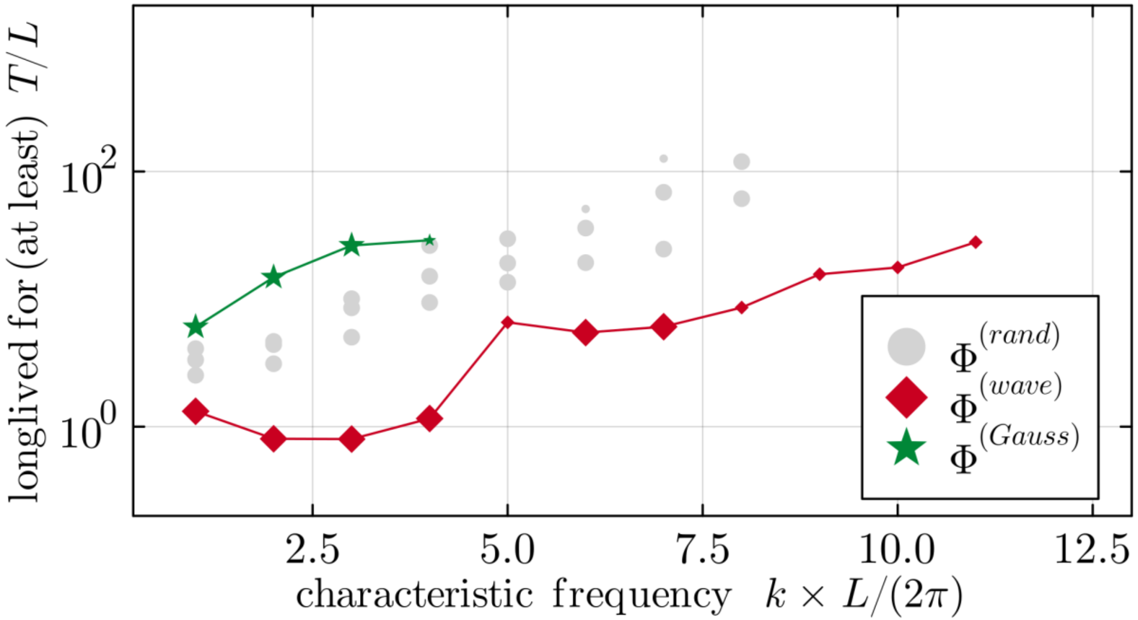

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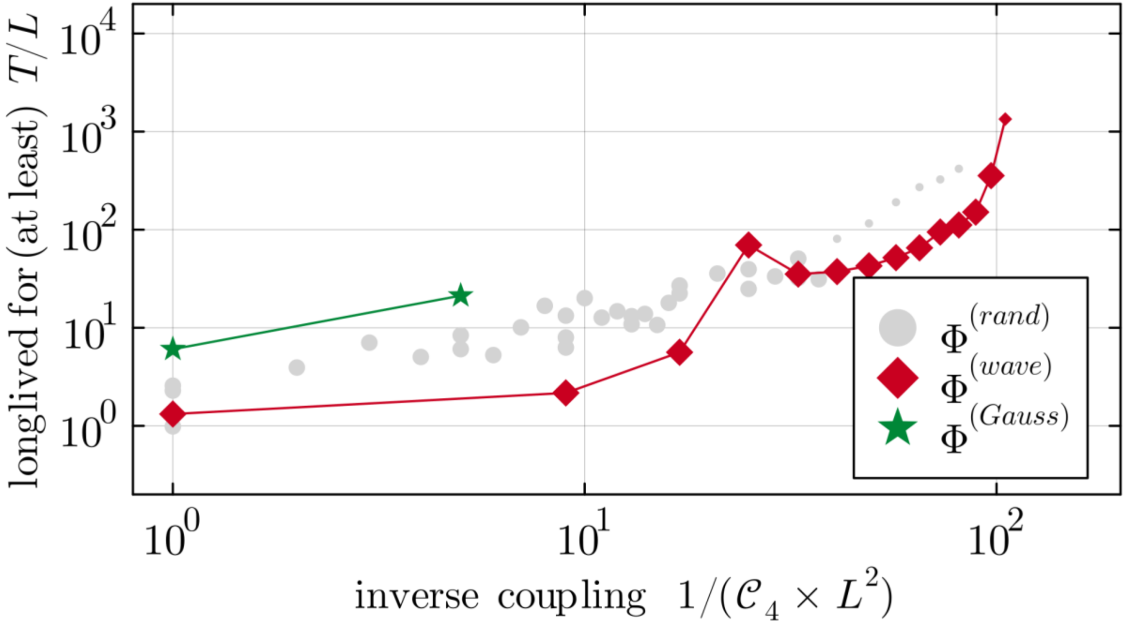

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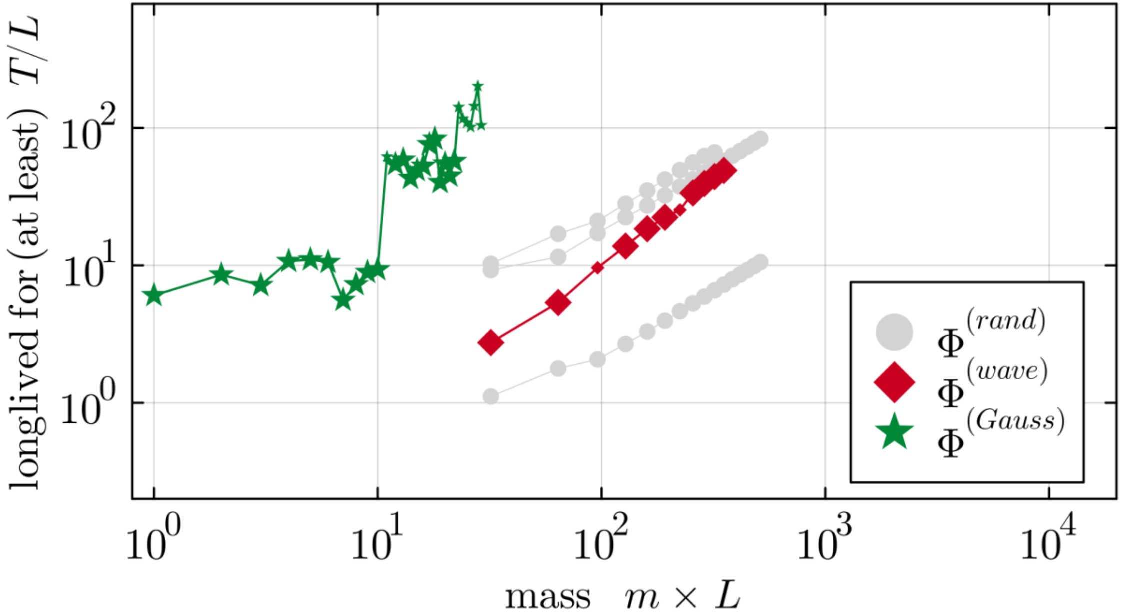

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Deffayet, Held, Mukohyama, Vikman, 2504.11437

... admit for (arbitrarily) long-lived motion.

Opposite-sign kinetic terms …

Deffayet, Held, Mukohyama, Vikman, 2504.11437

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- 4^{th} order FD
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Model parameters setup in nontrivial vacuum

$$m_\phi^2 \equiv m_\chi^2/15 \equiv m^2$$

Random initial data

Opposite-sign kinetic terms ...

Deffayet, Held, Mukohyama, Vikman, 2504.11437

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model parameters

Deffayet, Held, Mukohyama, Vikman, JCAP 11 (2023) 031

$$V_{LV}^{(4)}(x,y) = \left(\frac{m_\phi^2}{2} - \frac{m_\chi^2}{2}\right)(\phi^2-\chi^2)^2 + \mathcal{C}_4(\phi^4-\chi^4) + \mathcal{C}_4(\phi^2-\chi^2)^3$$

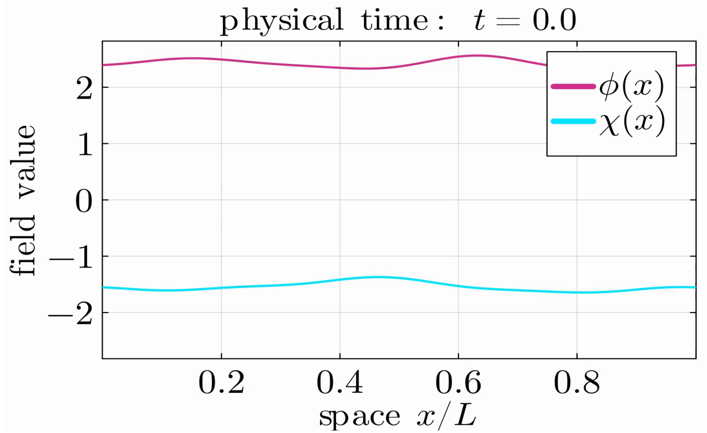
(1+1)D Simulation converges to the solution of the continuum field theory

- 4^{th} order FD
- 4^{th} order RK4 timestep
- self-convergence rate verified at all times

Model parameters setup in nontrivial vacuum

$$m_\phi^2 \equiv m_\chi^2/15 \equiv m^2$$

Random initial data



Deffayet, Held, Mukohyama, Vikman, 2504.11437 (see also JCAP 11 (2023) 031)

... can induce nontrivial long-lived vacua.

Opposite-sign kinetic terms …

$$V = \lambda \phi^2 \chi^2$$

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Model parameters

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- $m_\phi = 0$;
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Plane-wave initial data

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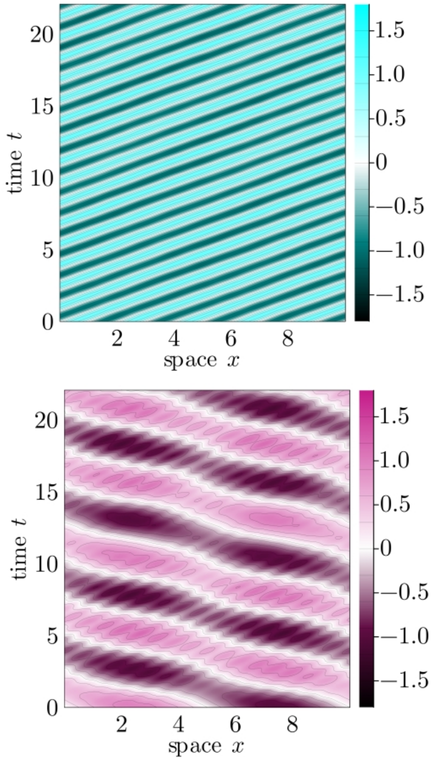

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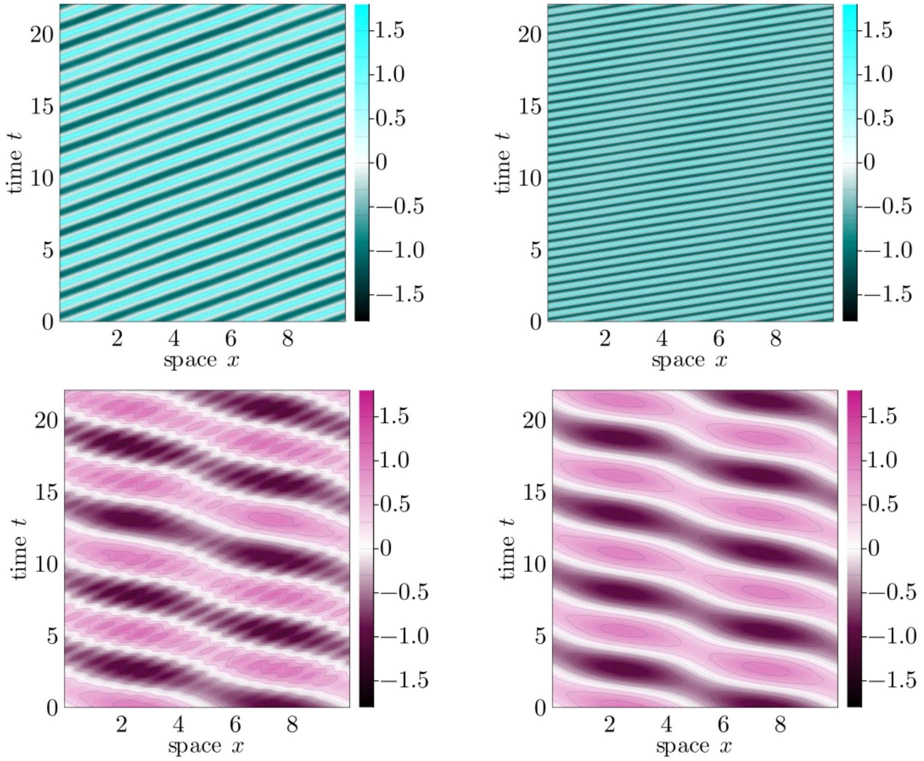

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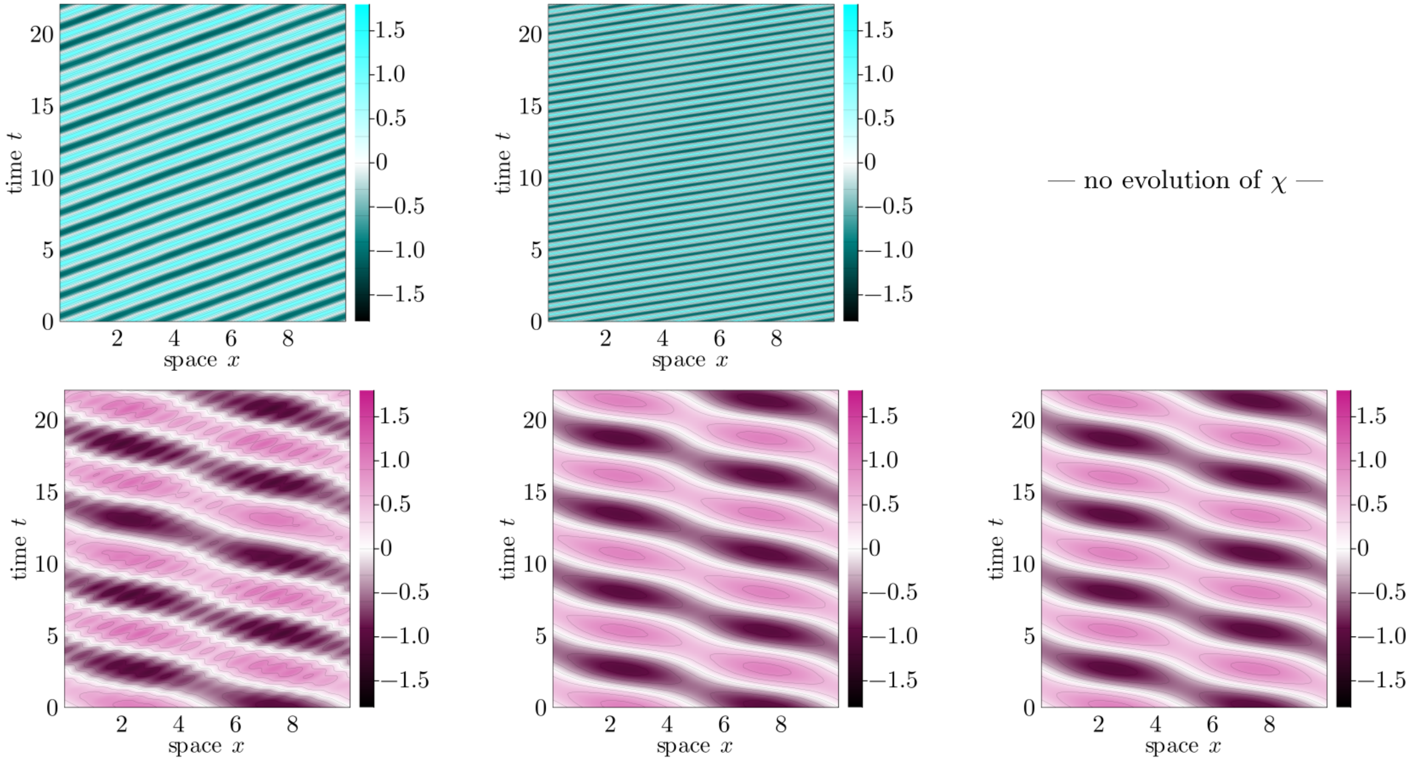

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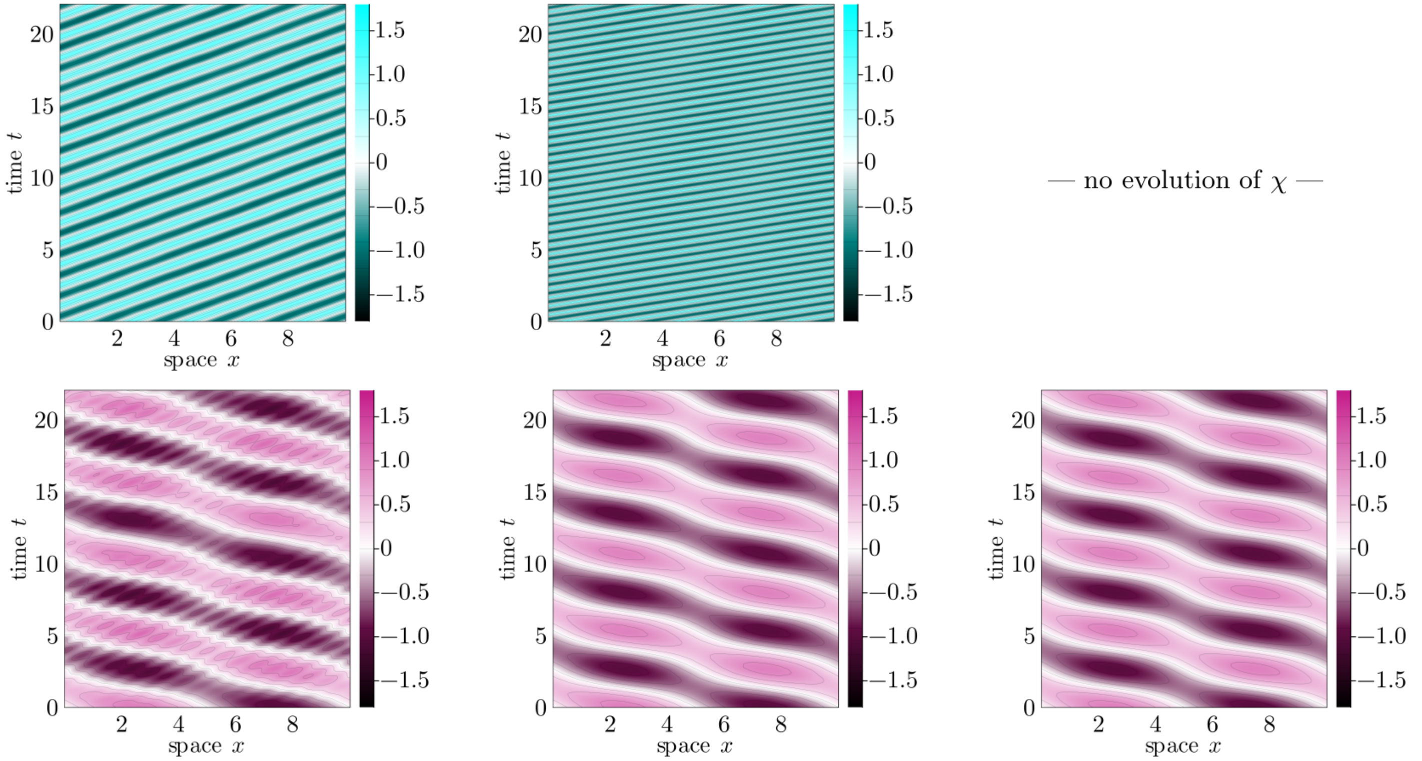
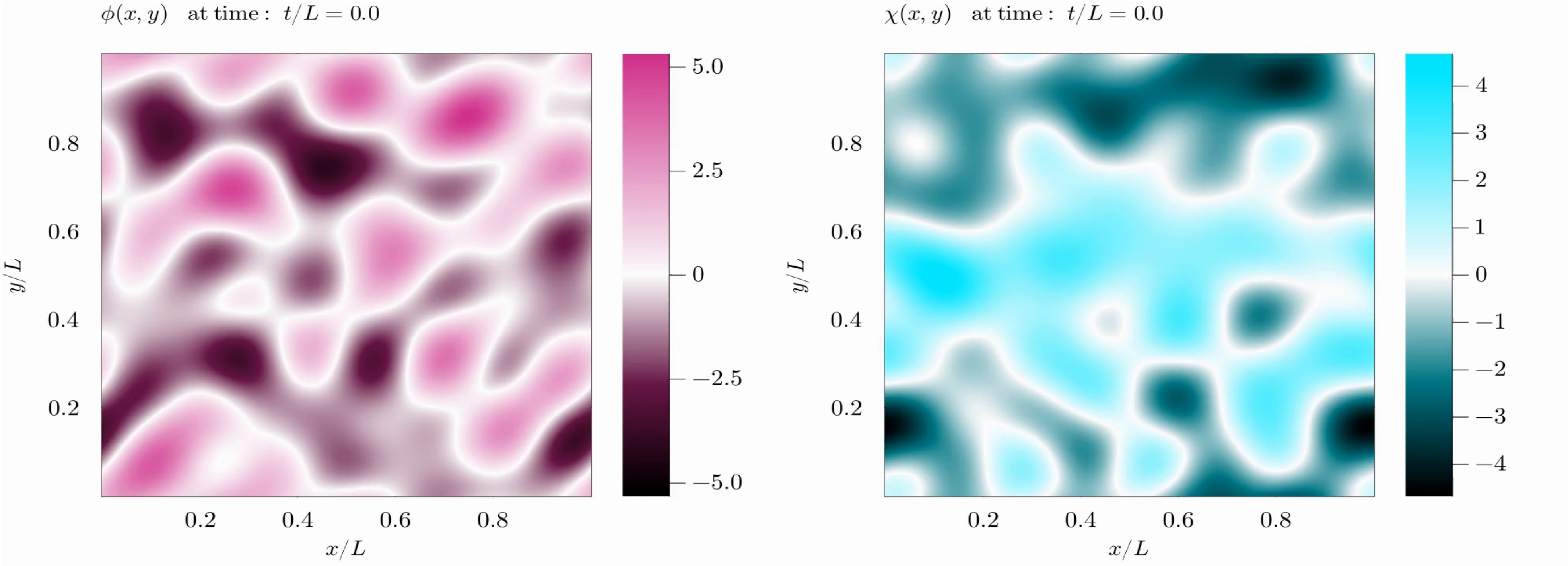


Deffayet, Held, Mukohyama, Vikman, 2504.11437 (see also Figueras, Kovács, Yao, 2505.00082)

… can effectively decouple if sufficiently heavy.



PRELIMINARY: results in (2+1) and spherically-symmetric scattering in (n+1) to appear soon

Part I: Ghostly interactions in classical field theory

The **Hamiltonian** of all higher-derivative non-degenerate classical point-particle theories is **unbounded from above and below.**

Ostrogradski 1857

Point-particle models can be stable if the potential at large phase-space distance is **dominated by stable self-interactions.**

Deffayet, Held, Mukohyama, Vikman, JCAP 11 (2023) 031

Classical field theories do not decay instantaneously and can **exhibit longlived motion.**

Deffayet, Held, Mukohyama, Vikman, 2504.11437

~~All non-degenerate **higher-derivative classical point-particle** theories **exhibit runaway** solutions.~~

~~Ok, but **field theories** will still **decay instantaneously** because of an infinite phase-space volume at high energy.~~

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Deffayet, Held, Mukohyama, Vikman, JCAP 11 (2023) 031
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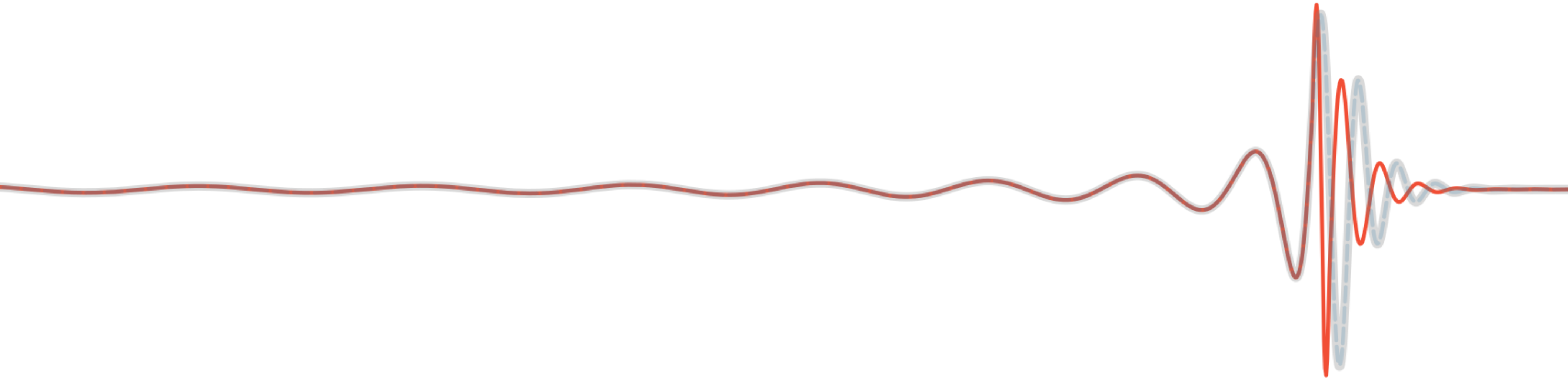
Ok, but **quantised field theories** will **decay instantaneously**, right?

Part I: Ghostly interactions in classical field theory

Deffayet, Held, Mukohyama, Vikman, JCAP 11 (2023) 031
Deffayet, Held, Mukohyama, Vikman, 2504.11437

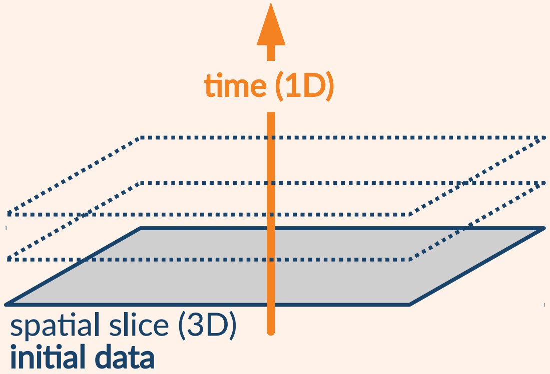


Part II: Nonlinear evolution & black hole binaries

Noakes, JMP 24, 1846 (1983);
Figueras, Held, Kovacs, 2407.08775

Held, Lim, PRD 104 (2021) 8
Held, Lim, PRD 108 (2023) 10
Held, Lim, 2503.13428

A well-posed initial value problem (IVP) ...



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- **exists** for all future time
- is **unique**
- and depends **continuous**ly on the initial data

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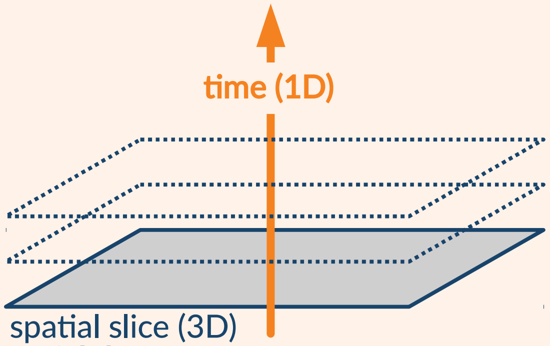

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Formal proof of existence and uniqueness
Yvonne Choquet-Bruhat ‘52

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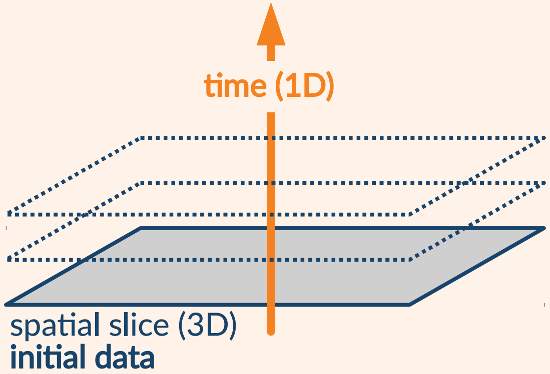

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Held, Lim ‘21, ‘23, ‘25; Cayuso ‘23;
East, Siemonsen ‘23

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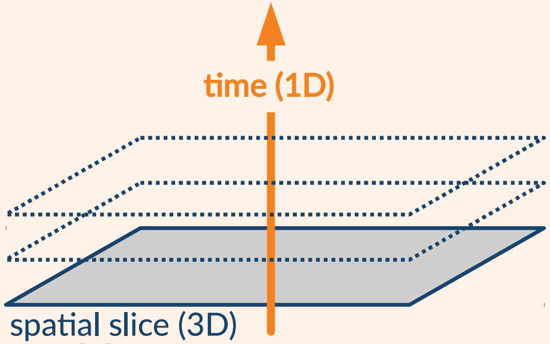

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… and for the EFT (at fixed order)

Figueras, Held, Kovacs, 2407.08775

Leray weights ...

Choquet-Bruhat, DeWitt-Morette, 1982

Let i be an index labeling a system of second order equations E_i for the variables v_i .

The principal part can be diagonalised if there exist positive integers s_i and t_i such that

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... give a prescription to diagonalise the principal part.

Leray weights: A simple example …

- System of PDEs:

$$\Box u = v$$

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... that can be diagonalised.

Leray weights: A simple example ...

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$$\Box v \overset{3|1}{=} \partial_t^2 u$$

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General Relativity (in harmonic gauge) ...

- Gauge potential: $F^a \equiv -g^{cd}\Gamma^a_{cd}$
- Ricci curvature: $R_{ab} \sim \Box g_{ab} + g_{c(a}\nabla_{b)}F^c + \mathcal{O}(g, \partial g) = 0$

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For constraint propagation see
Choquet-Bruhat ‘52

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For constraint propagation see
Choquet-Bruhat ‘52

… is already in wave-like form.

Quadratic Gravity ...

Held, Lim, PRD 104 (2021) 8

- recall $\mathcal{L} = M_{Pl}^2 \left[R + \frac{1}{12 m_0^2} R^2 + \frac{1}{4 m_2^2} C_{abcd} C^{abcd} \right]$

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Stelle, PRD 16 (1977) 953-969
Noakes, JMP 24, 1846 (1983)
Held, Lim, PRD 104 (2021) 8

2$^{\text{nd}}$-order variables

$$\Box g_{ab} \sim R_{ab} \equiv S_{ab} + \frac{1}{4} g_{ab} R$$

massless spin-2 (graviton)

$$\Box R = m_0^2 R$$

massive spin-0 (scalar)

$$\Box S_{ab} = -\frac{1}{3}\left(\frac{m_2^2}{m_0^2} - 1\right)(\nabla_a \nabla_b R) - 2\, S^{cd} C_{acbd} + \mathcal{O}_{\text{lower order}}$$

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2nd-order variables

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massless spin-2 (graviton)

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massive spin-0 (scalar)

$$\Box S_{ab} \overset{3|1}{=} -\frac{1}{3} \left(\frac{m_2^2}{m_0^2} - 1 \right) (\nabla_a \nabla_b R) - 2 S^{cd} C_{acbd} + \mathcal{O}_{\text{lower order}}$$

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Held, Lim, PRD 104 (2021) 8

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Cubic Gravity (after suitable field redefinitions) ...

Figueras, **Held**, Kovacs, 2407.08775

- recall $\mathcal{L}^{(3)}_{\mathrm{EFT}} = \frac{1}{M_{\mathrm{Pl}}^2}\left[\alpha_1\, R^{ab}\,\Box\, R_{ab} - \beta_1\, R\,\Box\, R + \gamma_3\, C_{ab}{}^{cd}\, C_{cd}{}^{ef}\, C_{ef}{}^{ab}\right]$

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order-reduced 2nd-order field equations

$$\Box g_{ab} \sim R_{ab} \equiv S_{ab} + \frac{1}{4} g_{ab}\, R$$

$$\Box C_{abde} = \mathcal{O}^{C}_{abde}(\partial C,\, \partial\partial S,\, \partial\partial R)$$

$$\Box R \equiv R^{(1)}$$

$$\Box S_{ab} \equiv S^{(1)}_{ab}$$

$$\Box R^{(1)} \equiv \mathcal{O}^{R}(\partial C,\, \partial\partial S,\, \partial\partial R)$$

$$\Box S^{(1)}_{ab} \equiv \left(1 - \frac{2\beta_1}{\alpha_1}\right)\left(\frac{1}{4} g_{ab}\,\Box - \nabla_a \nabla_b\right) R^{(1)} + \mathcal{O}^{S}_{ab}(\partial C,\, \partial\partial S,\, \partial\partial R)$$

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Higher order EFT (after suitable field redefinitions) …

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$$\Box C_{abde} = \mathcal{O}^C_{abde}(\partial C, \partial\partial S, \partial\partial R)$$

$$\Box R^{(k)} \equiv R^{(k+1)} \qquad \forall 0 \leq k < n$$

$$\Box S^{(k)}_{ab} \equiv S^{(k+1)}_{ab} \qquad \forall 0 \leq k < n$$

$$\Box R^{(n)} \equiv \mathcal{O}^R(\partial^{n-1} C, \partial^{n-k} S^{(k)}, \partial^{n-k} R^{(k)})$$

$$\Box S^{(n)}_{ab} \equiv \mathcal{O}^S_{ab}(\partial^{n-1} C, \partial^{n-k} S^{(k)}, \partial^{n-k} R^{(k)})$$

Higher order EFT (after suitable field redefinitions) ...

Figueras, **Held**, Kovacs, 2407.08775

- Inductively, this extends to $\mathcal{L}^{(n)}_{\text{reg}} = \sum_{k=0}^{n} \left[\alpha_k R^{ab} \Box^k R_{ab} - \beta_k R \Box^k R \right]$ with $\alpha_n = 2\beta_n$

order-reduced 2nd-order field equations

$$\Box g_{ab} \sim R_{ab} \overset{2|0}{\equiv} S_{ab} + \frac{1}{4} g_{ab} R$$

$$\Box C_{abde} \overset{3|1}{=} \mathcal{O}^C_{abde}(\partial C, \partial\partial S, \partial\partial R)$$

$$\Box R^{(k)} \overset{k+2|k}{\equiv} R^{(k+1)} \qquad \forall 0 \leq k < n$$

$$\Box S^{(k)}_{ab} \overset{k+2|k}{\equiv} S^{(k+1)}_{ab} \qquad \forall 0 \leq k < n$$

$$\Box R^{(n)} \overset{n+2|n}{\equiv} \mathcal{O}^R(\partial^{n-1} C, \partial^{n-k} S^{(k)}, \partial^{n-k} R^{(k)})$$

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Figueras, **Held**, Kovacs, 2407.08775

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... admits wave-like 2^{nd} order field equations.

Higher order EFT (after suitable field redefinitions) ...

Figueras, **Held**, Kovacs, 2407.08775

- Inductively, this extends to $\mathcal{L}_{\text{reg}}^{(n)} = \sum_{k=0}^{n} \left[\alpha_k R^{ab} \Box^k R_{ab} - \beta_k R \Box^k R \right]$ with $\alpha_n = 2\beta_n$

order-reduced 2nd-order field equations

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Not altered if supplemented with an action that only adds to the omitted lower-order terms.

see
Figueras, **Held**, Kovacs, 2407.08775
for a complete proof

... admits wave-like 2nd order field equations.

Well-posed initial value formulation ...

$$\mathcal{L}^{(1)}_{EFT} = M_{Pl}^2\, R$$

✓ Choquet-Bruhat '52

$$\mathcal{L}^{(2)}_{EFT} = \left[\alpha_0 R_{ab} R^{ab} - \beta_0\, R^2\right]$$

order-by-order field redefinitions of the form

$$g_{ab} \to g_{ab} + c_1\, g_{ab}\, X + c_2\, X_{ab}$$

can remove any term containing Ricci variables

$$\mathcal{L}^{(3)}_{EFT} = \frac{1}{M_{Pl}^2}\left[\alpha_1\, R^{ab}\, \Box\, R_{ab} - \beta_1\, R\, \Box\, R + \gamma_3\, C_{ab}{}^{cd} C_{cd}{}^{ef} C_{ef}{}^{ab} + \delta_{3,1}\, C_{abcd} C^{abcd} R + \delta_{3,2}\, C_{abcd} R^{ac} R^{bd} + \delta_{3,3}\, R^b_a\, R^c_b\, R^a_c + \delta_{3,4}\, R_{ab} R^{ab} R + \delta_{3,5}\, R^3\right]$$

$$\mathcal{L}^{(4)}_{EFT} = \frac{1}{M_{Pl}^4}\left[\alpha_2\, R^{ab}\, \Box^2\, R_{ab} - \beta_2\, R\, \Box^2\, R + \gamma_{4,1}\, (C_{abcd}\, C^{abcd})^2 + \gamma_{4,2} (C_{abcd}\, {}^*C^{abcd})^2 + \dots\right]$$

Well-posed initial value formulation ...

$$\mathcal{L}^{(1)}_{\mathrm{EFT}} = M_{\mathrm{Pl}}^2\, R$$

✓ Choquet-Bruhat '52

$$\mathcal{L}^{(2)}_{\mathrm{EFT}} = \left[\alpha_0 R_{ab} R^{ab} - \beta_0\, R^2\right]$$

✓ Noakes, JMP 24, 1846 (1983)
Held, Lim '21, '23, '25

order-by-order field redefinitions of the form

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$$\mathcal{L}^{(3)}_{\mathrm{EFT}} = \frac{1}{M_{\mathrm{Pl}}^2}\Big[\alpha_1\, R^{ab}\,\Box\, R_{ab} - \beta_1\, R\,\Box\, R + \gamma_3\, C_{ab}{}^{cd} C_{cd}{}^{ef} C_{ef}{}^{ab}$$
$$+ \delta_{3,1}\, C_{abcd} C^{abcd} R + \delta_{3,2}\, C_{abcd} R^{ac} R^{bd} + \delta_{3,3}\, R^b_a\, R^c_b\, R^a_c + \delta_{3,4}\, R_{ab} R^{ab} R + \delta_{3,5}\, R^3\Big]$$

$$\mathcal{L}^{(4)}_{\mathrm{EFT}} = \frac{1}{M_{\mathrm{Pl}}^4}\Big[\alpha_2\, R^{ab}\,\Box^2\, R_{ab} - \beta_2\, R\,\Box^2\, R + \gamma_{4,1}\,(C_{abcd}\, C^{abcd})^2 + \gamma_{4,2}(C_{abcd}\, {}^{*}C^{abcd})^2 + \dots\Big]$$

Well-posed initial value formulation ...

$$\mathcal{L}^{(1)}_{\mathrm{EFT}} = M_{Pl}^2\, R$$ ✓ Choquet-Bruhat '52

$$\mathcal{L}^{(2)}_{\mathrm{EFT}} = \left[\alpha_0 R_{ab}R^{ab} - \beta_0\, R^2\right]$$ ✓

Noakes, JMP 24, 1846 (1983)
Held, Lim '21, '23, '25

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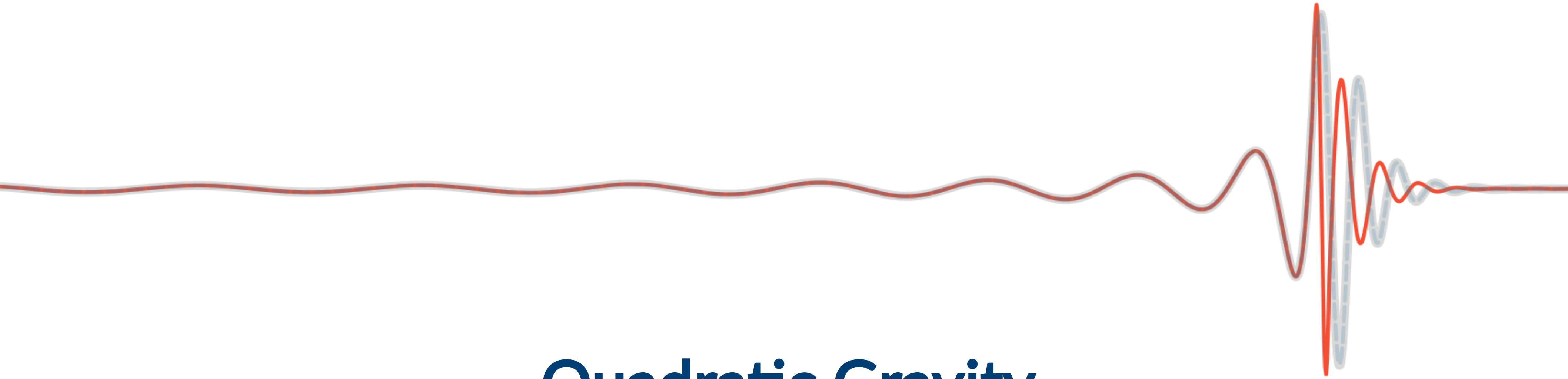
$$\mathcal{L}^{(3)}_{\mathrm{EFT}} = \frac{1}{M_{Pl}^2}\Big[\alpha_1\, R^{ab}\,\Box\, R_{ab} - \beta_1\, R\,\Box\, R + \gamma_3\, C_{ab}{}^{cd} C_{cd}{}^{ef} C_{ef}{}^{ab}$$ ✓

$$+\, \delta_{3,1}\, C_{abcd}C^{abcd}R + \delta_{3,2}\, C_{abcd}R^{ac}R^{bd} + \delta_{3,3}\, R^b_a\, R^c_b\, R^a_c + \delta_{3,4}\, R_{ab}R^{ab}R + \delta_{3,5}\, R^3\Big]$$

$$\mathcal{L}^{(4)}_{\mathrm{EFT}} = \frac{1}{M_{Pl}^4}\Big[\alpha_2\, R^{ab}\,\Box^2\, R_{ab} - \beta_2\, R\,\Box^2\, R + \gamma_{4,1}\,(C_{abcd}\, C^{abcd})^2 + \gamma_{4,2}(C_{abcd}\, {}^*C^{abcd})^2 + \ldots\Big]$$ ✓

Figueras, Held, Kovacs, 2407.08775

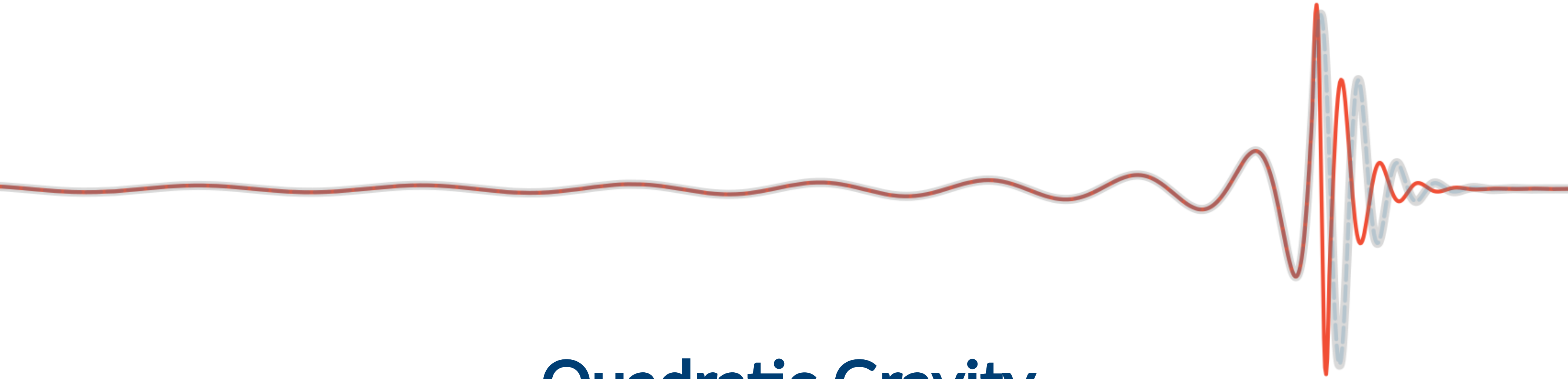
... of general effective field theories of gravity.



Quadratic Gravity

$$S = \int d^4x \sqrt{|g|}\, M_{\text{Planck}}^2 \left[R + \frac{1}{12 m_0^2} R^2 + \frac{1}{4 m_2^2} C_{abcd} C^{abcd} \right]$$

massless spin-2 massive spin-0 massive spin-2



Quadratic Gravity

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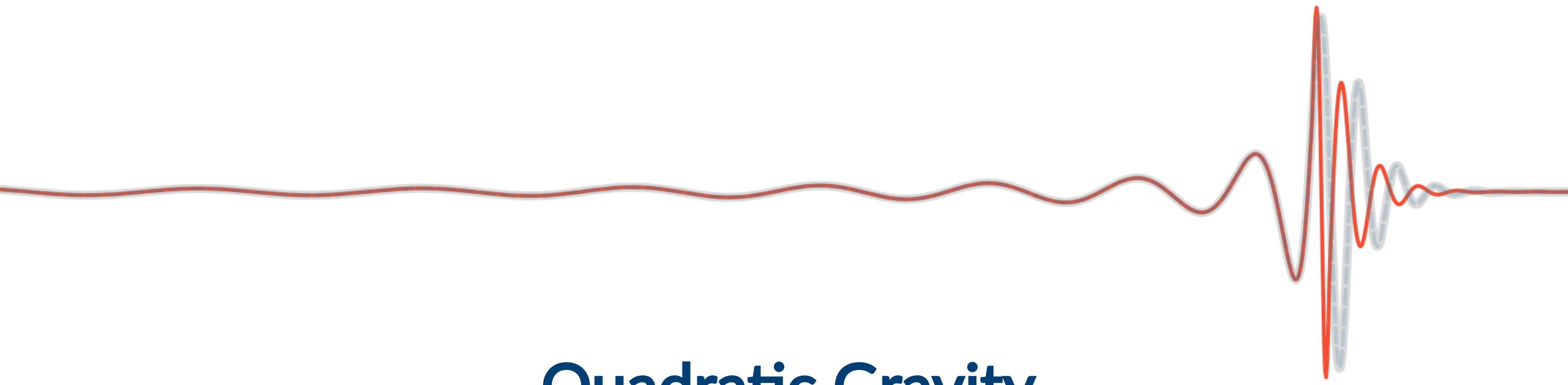
massless spin-2 massive spin-0 massive spin-2

as a benchmark model to show that heavy ghosts decouple

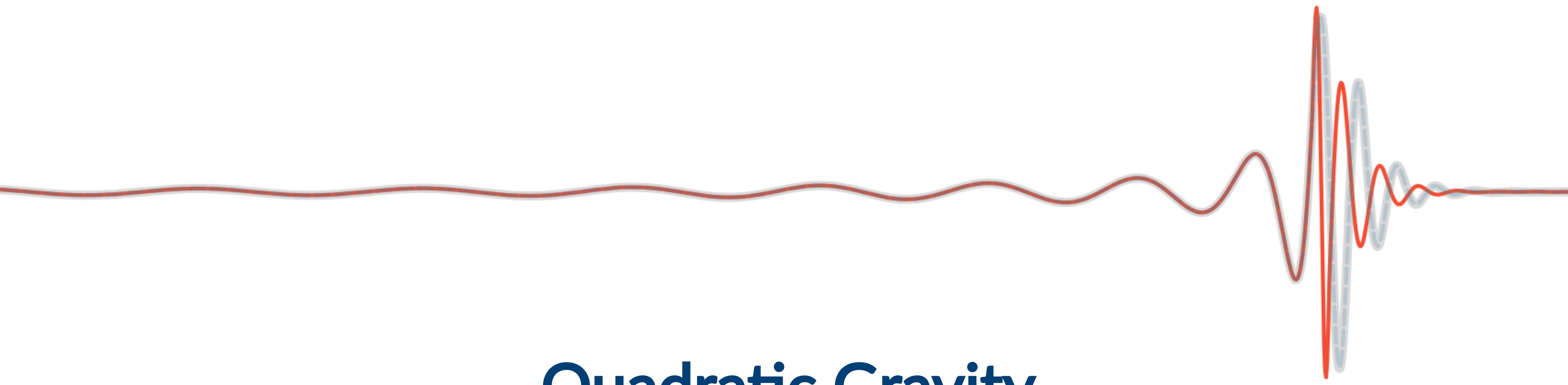


Quadratic Gravity

$$S = \int d^4x \sqrt{|g|}\, M_{\text{Planck}}^2 \left[R + \frac{1}{12 m_0^2} R^2 + \frac{1}{4 m_2^2} C_{abcd} C^{abcd} \right]$$

massless spin-2 massive spin-0 massive spin-2

as the leading-order terms before field redefinitions / in non-vacuum situations



Quadratic Gravity

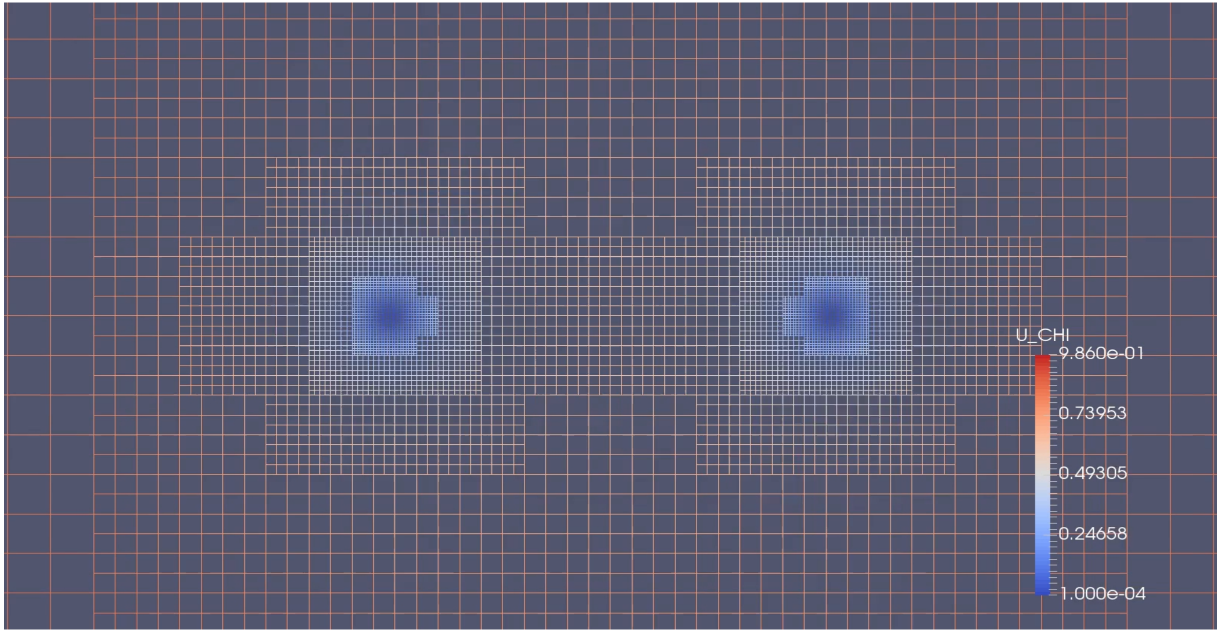
$$S = \int d^4x \sqrt{|g|}\, M^2_{Planck} \left[R + \frac{1}{12m_0^2} R^2 + \frac{1}{4m_2^2} C_{abcd} C^{abcd} \right]$$

massless spin-2 massive spin-0 massive spin-2

as a fundamental theory of gravity

Stelle, PRD 16 (1977) 953-969
Avramidi, Barvinsky, PLB 159 (1985) 269-274
Donoghue, Menezes, PRD 104 (2021) 4

High-performance computing …



- 4^{th} order finite differencing
- 4^{th} order Runge-Kutta timestepping
- adaptive mesh refinement
- parallelized on HPC clusters

Dendro-GR https://github.com/paralab/Dendro-GR

… required to solve the full (3+1) binary problem.

Black-hole binaries ...

EFT regime of validity

$GM\, m_2 \gg 1$	Held, Lim, PRD 108 (2023) 10 **no deviations**
$GM\, m_2 \sim 1$	**quantitative deviations**
$GM\, m_2 \lesssim 1$	**qualitative deviations** Held, Lim, 2503.13428

Waveforms for $G M\, m_2 \lesssim 0.43$ ...

Held, Lim, 2503.13428

QG masses		Binary parameters			
$G\, m_0\, M_2$	$G\, m_2\, M_2$	$\sqrt{G}\, M_1$	$q = \frac{M_1}{M_2}$	$a_{z,1}$	$a_{z,2}$
1	0.2	1	1	0	0

Waveforms for $GM\, m_2 \lesssim 0.43$ ...

Held, Lim, 2503.13428

QG masses		Binary parameters			
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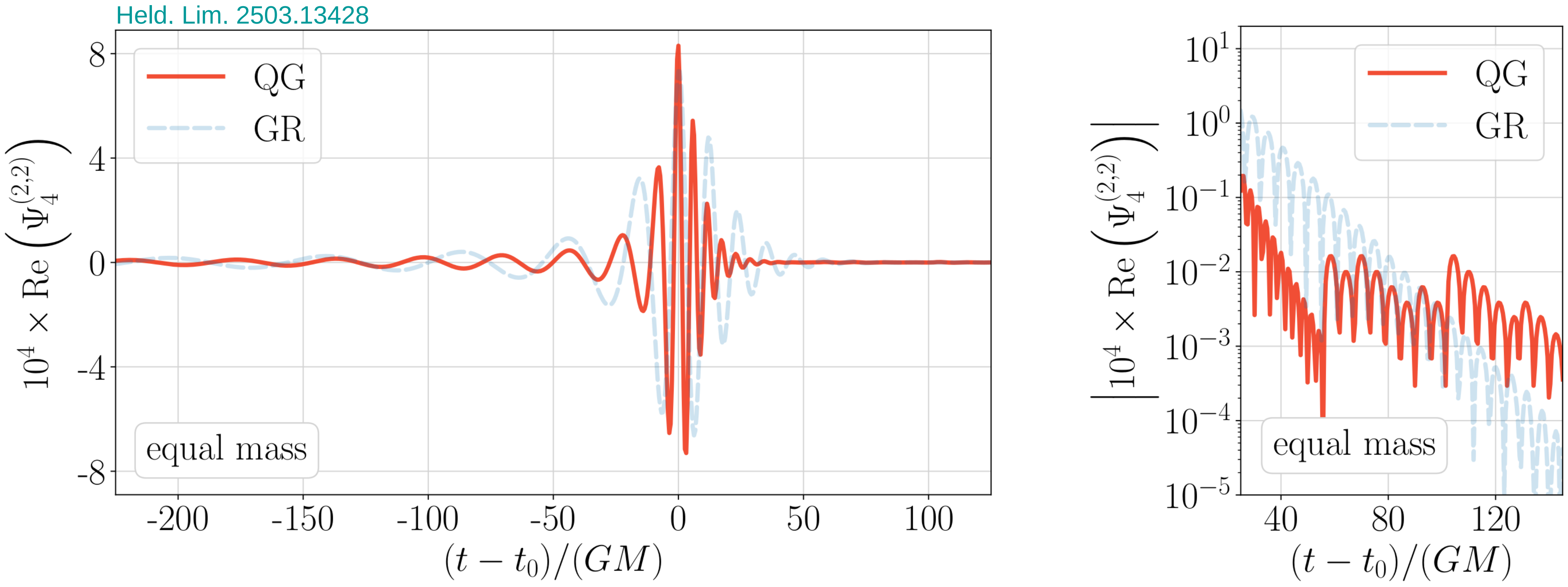
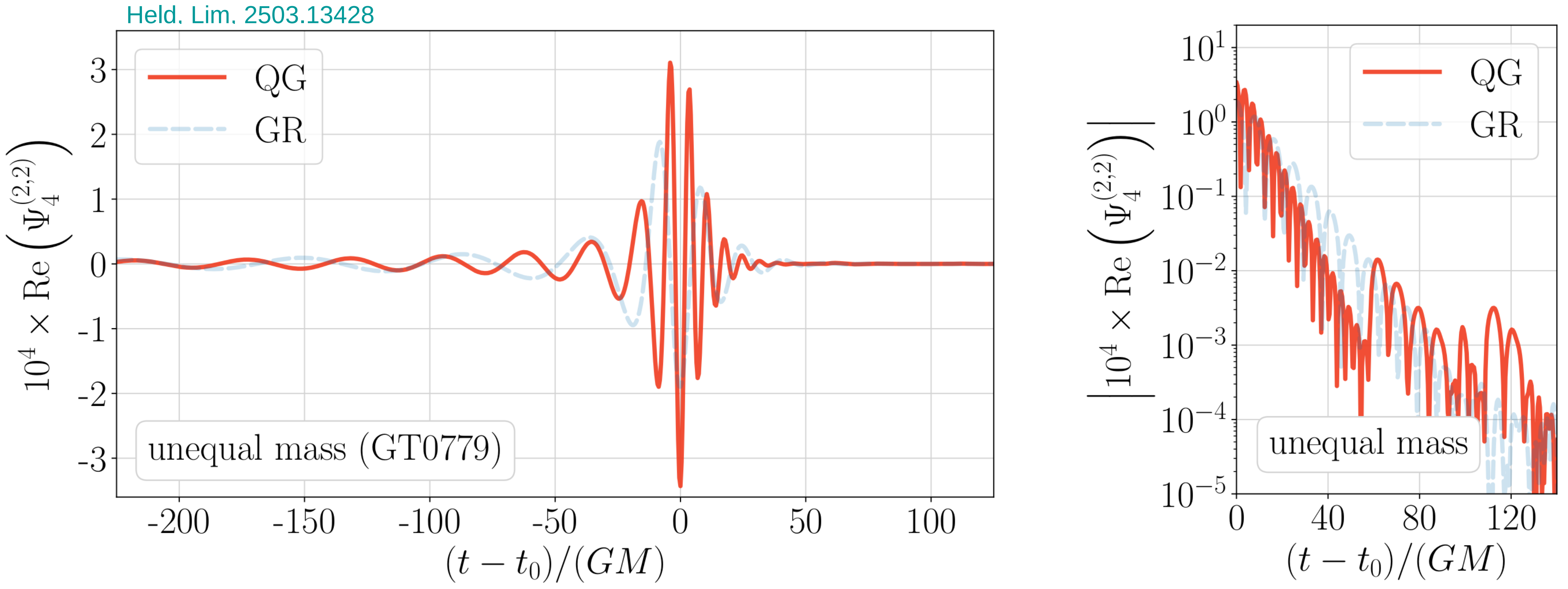


... deviate quantitatively.

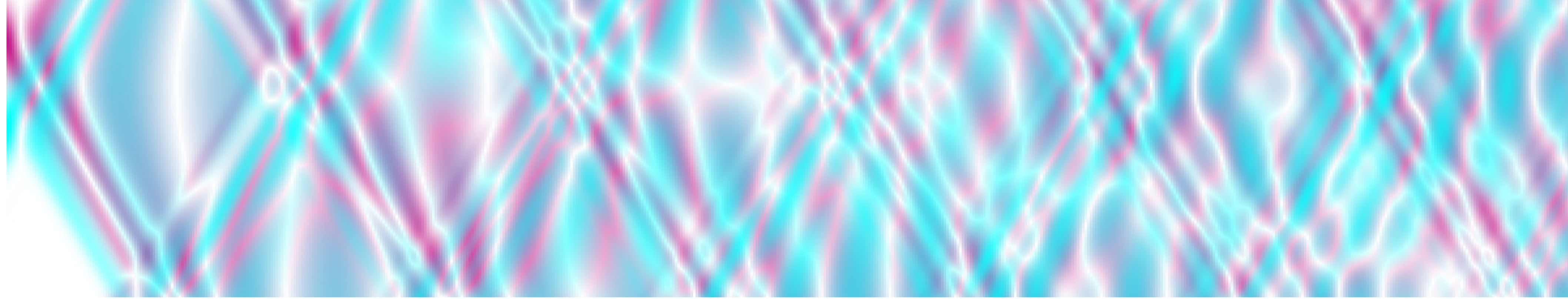
Waveforms for $GM\, m_2 \lesssim 0.43$ …

Held, Lim, 2503.13428

QG masses		Binary parameters			
$G\, m_0\, M_2$	$G\, m_2\, M_2$	$\sqrt{G}\, M_1$	$q = \frac{M_1}{M_2}$	$a_{z,1}$	$a_{z,2}$
1	0.2	1	5	-0.696	0



… deviate qualitatively.



Classical field theories with ghosts can be longlived.

Ghosts enable well-posed time evolution.

Access to the nonlinear regime of higher-derivative theories.